Dartmouth College

Mathematics 101

Homework 4 (due Wednesday, October 20)

1. For $n \geq 3$, characterize the center of the symmetric group S_n.

2. For $n \geq 5$, show that the only normal subgroups of S_n are $\{e\}$, A_n, and S_n. Use this to provide a proof different than Lang's that S_n is not solvable for $n \geq 5$.

3. For p and q distinct primes, show that any group of order $p^2 q$ is solvable.

4. Let G be a group of order 12, and assume that G has more than one Sylow 3-subgroup. Show that $G \cong A_4$. Hint: Letting G act on the set of Sylow 3-groups with provide a homomorphism $\varphi : G \to S_4$.