MATH 101: ALGEBRA I
WORKSHEET, DAY #10
"RINGS EXTRAVANGANZA"

Problem 1. What is the noncommutative ring with the smallest cardinality?

Problem 2. Show that no commutative ring has additive group isomorphic to $\mathbb{Q}/\mathbb{Z}$.

Problem 3. Let R be the ring of all continuous functions on $[0, 1]$.

(a) Observe that the collection I of functions $f \in R$ such that $f(1/3) = f(1/2) = 0$ is an ideal. Show this ideal is not prime.

(b) What are the maximal ideals of R?

Problem 4. Let k be a field and let R be a (commutative integral) domain that is a finite-dimensional k-algebra. Show that R is a field. Conclude that a finite domain is a field.

Problem 5. Let F be a field, let S be a ring, and let $n \geq 1$. Show that any ring homomorphism $\mathrm{M}_n(F) \to S$ is either zero or injective.

Date: Tuesday, 27 September 2016.