Problem 11.1. Let $V$ be an $F$-vector space, and let $\phi: V \to V$ be an $F$-linear map. We say that $\phi$ is an involution if $\phi^2 = 1$.

Suppose that $V$ is an inner product space and let $\phi$ be an involution on $V$. Show that the following are equivalent:

(a) $\phi$ is normal;
(b) $\phi$ is unitary;
(c) $\phi$ is self-adjoint; and
(d) All singular values of $\phi$ are equal to 1.