Problem 16.1. Let $R$ be a commutative ring.

(a) Let $A, B$ be $R$-algebras (with 1). Show that there exists a unique structure of $R$-algebra on the $R$-module $A \otimes_R B$ such that

$$(\alpha \otimes \beta) \cdot (\alpha' \otimes \beta') = \alpha \alpha' \otimes \beta \beta'$$

for all $\alpha, \alpha' \in A$ and $\beta, \beta' \in B$.

(b) Let $A$ be an $R$-algebra, let $S$ be a commutative ring, and let $f: R \to S$ be a ring homomorphism. Equip $S$ with the structure of $R$-module via $f$. (Write out what this means.) Show accordingly that $A \otimes_R S$ can be given the structure of an $S$-algebra.

(c) Describe the $\mathbb{C}$-algebras $\mathbb{C} \otimes_R \mathbb{C}$ and $\mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$.

Date: Assigned Monday, 16 October 2017; due Wednesday, 18 October 2017.