Problem 25.1. Let $M$ be the $\mathbb{Z}$-module generated by $x_1, x_2, x_3, x_4$ subject to the relations
\[
\begin{align*}
x_1 + 3x_2 - 9x_3 &= 0 \\
x_1 + 3x_2 + 3x_3 + 12x_4 &= 0 \\
2x_1 + 4x_2 + 2x_3 + 24x_4 &= 0
\end{align*}
\]
Give an explicit isomorphism of $M$ to a direct sum of cyclic abelian groups. What are the invariant factors and elementary divisors of $\text{Tor}(M)$?

Problem 25.2. Let $R$ be a PID. Let $a, b \in R$, not both zero. Write $(a, b) = (g)$ for $g \in R$, so that there exist $x, y \in R$ such that $ax + by = g$. Show that $(x, y) = R$. 

\textit{Date:} Assigned Friday, 3 November 2017; due Monday, 6 November 2017.