

Math 73/103 Homework week 1

Last Updated: September 16, 2021

Monday 09/13/2021

1. Suppose (X, ρ_X) and (Y, ρ_Y) are metric spaces. Show that

$$\rho((x_1, y_1), (x_2, y_2)) := \rho_X(x_1, x_2) + \rho_Y(y_1, y_2)$$

defines a metric on the Cartesian product $X \times Y$. Find another metric on $X \times Y$.

2. Suppose ρ is a pseudometric on X . Show that $\rho(x, y) = 0$ defines an equivalence relation, and that ρ is a metric on the set \tilde{X} of equivalence classes under this relation.
3. Let (X, ρ) be a metric space. Show that

$$\tilde{\rho}(x, y) := \frac{\rho(x, y)}{1 + \rho(x, y)}$$

defines a metric on X . Show that X is bounded in this metric. Recall, a set $A \subset X$ is bounded in a metric ρ if $\delta(A) < \infty$, where $\delta(A) := \sup_{x, y \in A} \rho(x, y)$ is the diameter of A .

(Hint: To show that $\tilde{\rho}$ satisfies the triangle inequality, consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(t) = \frac{t}{1+t}$. Show this function is monotone increasing and then apply this to $\tilde{\rho}$.)

4. Every normed space has an associated metric. Does the converse hold? Give a proof or a counter example.