Math 73/103 Homework week 2

Last Updated: September 20, 2021

Monday 09/20/2021

1. Show that if (x_n) and (y_n) are Cauchy sequences in a metric space (X, ρ), then $\rho(x_n, y_n)$ converges.

2. Suppose (X, ρ_X) is a metric spaces and $(x_n) \subset X$ is a Cauchy sequence. Show that if (x_n) has a convergent subsequence then (x_n) is convergent.

3. Let (X, ρ_X) be a metric space. We say $x \in X$ is a cluster point of a sequence (x_n) if $B_r(x)$ contains and infinite number of points from (x_n) for every $r > 0$. Show that if (x_n) is Cauchy and x is a cluster point, then (x_n) converges.

4. Suppose (X, ρ_X) is a metric space. Show that the uniform limit of continuous functions $f_n : X \to \mathbb{C}$ is continuous.

5. Let X be the metric space of all real sequences $x = (x_n)$ such that only finitely many x_i are non-zero, with metric given by

$$\rho(x, y) = \sum_i |x_i - y_i|.$$

   Note these sums are finite, but the number of summands depends on x and y. Let $(x_n) \subset X$ be a sequence such that $x_i^{(n)} = i^{-2}$ for $i = 1, 2, \ldots, n$ and $x_i^{(n)} = 0$ for $i > n$. Show that (x_n) is Cauchy, but not convergent.