Course name: Math 106: Stochastic Processes with Applications

Instructor: Feng Fu

ORC Description: Stochastic models are central to the study of many problems in physics, engineering, finance, evolutionary biology, and medicine. This course introduces concepts and techniques in probability theory and key methods for stochastic processes, along with their applications to the natural sciences.

Textbook: The Elements of Stochastic Processes, Norman T. J. Bailey, John Wiley & Sons, Inc. (1963)

Syllabus

Week 1: Basic concepts of probability & generating function approach
- Introduction & examples
- Definitions & elementary results
- Generating functions I

Week 2: Applications of generating function approach
- Generating functions II
- Simulating stochastic dynamics: Gillespie algorithm
- Spreading dynamics on graphs & networks

Week 3: Markov chains
- Transition matrices, classification of states of a Markov chain & recurrence
- Recurrent Markov chains & limit theorems
- Martingales

Week 4: Random walks
- Discrete-time random walks & Gambler's ruin
- Moran process & fixation time
- Coalescent theory

Week 5: Continuous time Markov processes
- Continuous-time random walks
- Chapman-Kolmogorov equations
- Poisson process & Random-variable technique

Week 6: Diffusion theory
- Diffusion approximation
- Moran process in large populations
- Absorption time & fixation time

Week 7: Birth and death processes
- Homogeneous birth and death processes
- The effect of immigration
- General multiplicative processes

Week 8: Branching processes
- Discrete branching processes
- Generating function approach & extinction probabilities
- Multi-type branching processes