Math 74/114, Spring 2017

Homework set 1, due Wed Apr 5

This homework set is due on Wednesday April 5, at the start of class. Discussion of the problems is permitted, and even recommended. But you should write up and hand in your own solutions.

1. Do Hatcher p.18 exercises 1, 3ab.

2. Every manifold is a CW complex (i.e., can be given a cell structure).

   (a) Describe the skeletons $X^0 \subset X^1 \subset X^2$ of a CW structure for $\mathbb{R}^2$.

   (b) According to p.6 example 0.6, $\mathbb{C}P^2$ has a cell structure with one 0-cell, one 2-cell, one 4-cell. Explicitly describe the two attaching maps.

3. Consider a graph G with one vertex and a countable infinity of edges (loops). G has two natural topologies. First, G is a CW complex with one 0-cell and infinitely many 1-cells. Secondly, G is a metric space. Assume that each edge has length one, and the distance between two points is by definition the length of the shortest path between them. Prove that the topology of G as a CW complex is not the same as the topology of G as a metric space.

4. Give an explicit formula for a deformation retract of $\mathbb{R}^2 \setminus \{0\}$ onto the unit circle S^1.

5. Let S be a surface of genus g (i.e. a closed surface with g 'holes') that is embedded in $\mathbb{R}^3$. Let X be the unbounded connected component of $\mathbb{R}^3 \setminus S$ (the region 'outside' of S). Identify a wedge sum of circles and spheres that is homotopy equivalent to X. (How many circles and spheres do you need?)

   Explain by which sequence of elementary steps (deformation retract, collapsing a subspace, attaching a space) you get from X to your wedge sum. You do not have to rigorously define the various maps.