Math 74/114, Spring 2017

Homework set 2, due Wed Apr 12

This homework set is due on Wednesday April 12, at the start of class. Discussion of the problems is permitted, and even recommended. But you should write up and hand in your own solutions.

1. Do problems 10, 20 in section 1.1 (p.38-39) of Hatcher's book.

2. Do problem 6 in section 1.1 (p.38). Then answer the following question. Assuming X is path-connected, when is $\Phi : \pi_1(X, x_0) \to [S^1, X]$ a bijection?

3. Do problem 18 and 18a in section 1.1 (p.39). Skip 18b.

4. Which of the following two contradictory statements is false? Explain what is wrong with the argument.

   - Since $\pi_1(S^1) \cong \mathbb{Z}$ it follows that S^1 is not contractible.
   - By definition $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. The identity map $f_0 = \mathbb{1}$ is homotopic to the retract $f_1(x, y) = (1, 0)$ via the homotopy $f_t(x, y) = (x - tx + t, y - ty)$, with $t \in I$ as usual. So S^1 deformation retracts to a point.

5. Let X be a solid torus, and $T = \partial X$ its boundary. Prove that there is no retract $r : X \to T$. (More precisely, let T be a torus embedded in $\mathbb{R}^3$, and X the closure of the bounded component of $\mathbb{R}^3 \setminus T$.)

6. What is the fundamental group of the space X obtained by removing a solid torus $S^1 \times D^2$ from S^3. (*Hint:* Think of S^3 as $D^3 \cup CS^2$, i.e., the ball D^3 with a cone on its boundary $S^2 = \partial D^3$.)