Math 74/114, Spring 2017

Homework set 5, due Wed May 10

This homework set is due on Wednesday May 10, at the start of class. Discussion of the problems is permitted, and even recommended. But you should write up and hand in your own solutions.

1. Choose a Δ-complex structure for S^2 and calculate the simplicial homology groups $H_n^\Delta(S^2)$.

2. Do Hatcher's Exercises 8, 11, 12 from section 2.1, p. 131.

3. Let M_g be a genus g surface.

   (a) What is the abelianization of $\pi_1(M_g)$?

   (b) Choose a Δ-complex structure for M_2 and calculate $H_1^\Delta(M_2)$.

   (c) How do the generators of the abelianization of $\pi_1(M_g)$ correspond to the generators of $H_1^\Delta(M_2)$?

4. Let $K = \mathbb{R}/n\mathbb{Z}$. Note that K is homeomorphic to a circle. Recall that $\Delta^1 = \{(t_0, t_1) \in \mathbb{R}^2 \mid t_0 + t_1 = 1, t_i \geq 0\}$. Associated to a continuous map $f : K \to X$ is a singular 1-chain

$$\xi_f = \sigma_0 + \sigma_1 + \cdots + \sigma_{n-1} \qquad \sigma_i : \Delta^1 \to X,\ \sigma_i(t_0, t_1) = f(i + t_1)$$

   (a) Show that ξ_f is a 1-cycle.

   (b) Prove that $f : K \to X$ and $g : K \to X$ are homotopic, then ξ_f is homologous to ξ_g, i.e. $[\xi_f] = [\xi_g] \in H_1(X)$.