Dartmouth College

Mathematics 115 — Practice Problems 3

1. Let $|\cdot|$ be a non-archimedean valuation on a field F. Given three distinct points $\alpha, \beta, \gamma \in F$, show that the "triangle" they determine is isosceles. Here the length of the side between α and β is $|\alpha - \beta|$.

2. Let $|\cdot|$ be a non-archimedean valuation on a field F. For $\epsilon > 0$ a real number, and $\alpha \in F$, let $B_\epsilon(\alpha) = \{\gamma \in F : |\gamma - \alpha| < \epsilon\}$. Let $\beta \in B_\epsilon(\alpha)$. Show that $B_\epsilon(\alpha) = B_\epsilon(\beta)$, that is every point in an open disk is the center of the disk.

3. Let $|\cdot|$ be a valuation on a field F. Show that for any real number $0 < \rho \leq 1$, $|\cdot|^\rho$ is also a valuation. Show that this can be false if $\rho > 1$.

4. Determine the canonical 3-adic representation of $-\dfrac{13}{8}$. To be sure you are on the right track, it has the form
$$-\frac{13}{8} = 1 + 1 \cdot 3 + 2 \cdot 3^2 + \sum_{k=3}^{\infty} c_k 3^k.$$