MATH 11: MULTIVARIABLE CALCULUS
MIDTERM 1 REVIEW

Problem 1. Let $w = xy + yz + xz$ and

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = r\theta.$$

Using the chain rule, compute $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$, written as functions of r and θ.

Date: Wednesday, November 19.

Problem 2. Let $P_0 = (-1, -1, 0)$, $P_1 = (2, 2, 1)$, $P_2 = (3, 0, 1)$.

(a) Find the area of the space triangle with vertices P_0, P_1, P_2.

(b) Find the equation of the plane containing P_0, P_1, P_2.

(c) Does the plane in (b) intersect the line in the direction of $\langle 1, -2, 1 \rangle$ through the origin? If so, find the point of intersection.

Problem 3.

(a) Compute the limit or show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}.$$

(b) Compute the limit or show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}.$$

Problem 4.

(a) Parametrize the curve C given by the intersection of the surfaces $y = x^2$ and $z = xy$.

(b) Sketch the level curves for the function $z = xy$.

(c) Find a parametric equation of the tangent line to C at $(1, 1, 1)$.

(d) Estimate the arc length on C between $(0, 0, 0)$ and $(1, 1, 1)$ to a nearest integer.

Problem 5. The gas law for an ideal gas is

$$PV = mRT$$

where P is the pressure, V is the volume, m is the mass, R is a constant, and T is the temperature. Prove that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1.$$

Problem 6. Find a point $P = (x_0, y_0, z_0)$ on the surface

$$z = f(x, y) = x^2 + 3xy$$

such that the tangent plane to the surface at P is parallel to the plane $2x + 3y + 4z = 5$.