MATH 11: MULTIVARIABLE CALCULUS
MIDTERM 2 REVIEW

Problem 1. Short answers.

(a) Let $(\nabla f)(1,1) = \langle 3, -5 \rangle$. What are the signs of the directional derivative of f at $(1,1)$ in the direction $\nwarrow$ and $\downarrow$?

(b) True or false: the function $f(x,y) = x^2y + 4xy + 4y$ has a local maximum at the origin.

(c) Maximize x^2y^2 subject to $x^2 + y^2 = 4$.

Date: Monday, October 24.

Problem 2. Evaluate the integral

$$\iint_R x^3 \sin(x^2 y) \, \mathrm{d}A$$

over the rectangle R with $0 \leq x \leq \sqrt{\pi/2}$ and $0 \leq y \leq 2$.

Problem 3. Find the local maxima and minima of $f(x, y) = y^2 - 2y \cos x$.

Problem 4. What is the average value of the function $f(x, y) = xy$ on the region bounded by the curves $y = 8\sqrt{x}$ and $y = x^2$?

Problem 5. Let D be the circle of radius a. Compute $\iint_D e^{-x^2-y^2}\,\mathrm{d}A$. What is the limit as $a \to \infty$?

Problem 6. Rewrite the integral

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} f(r,\theta,z) r \, \mathrm{d}z \, \mathrm{d}r \, \mathrm{d}\theta$$

in the orders $\mathrm{d}r \, \mathrm{d}z \, \mathrm{d}\theta$ and $\mathrm{d}\theta \, \mathrm{d}z \, \mathrm{d}r$, and convert to spherical coordinates (in any order).

Problem 7. Consider the region R bounded on 4 sides by the curves

$$xy = 1,\ xy = 4,\ x = y, \text{ and } y = 3x.$$

Sketch the region R. Under the transformation T defined by $u = xy$, $v = y/x$, what does the region R become? Use this change of coordinates to evaluate

$$\iint_R e^{xy}\, \mathrm{d}A.$$

Problem 8. Let $f(x, y) = \log(1 + xy)$.

(a) Compute the gradient of f at $P = (2, 3)$.

(b) Find a unit vector in the direction of steepest ascent for f at P; what is the maximal rate of change?

(c) Find the unit vectors in the direction of no change for f at P.

(d) Sketch the level curves of f in the first quadrant (including the point P) and the unit vectors in (b) and (c).