MATH 11: MULTIVARIABLE CALCULUS
WEEK 5 REVIEW

Mark the following true or false. If not otherwise specified, $f : \mathbb{R}^2 \to \mathbb{R}$ is a function.

Problem 1. A figure 8 curve cannot occur as the level curve of a function f.

Problem 2. The function $f(x,t) = \exp(x-t)$ is a solution of the equation $f_{xx} = f_{tt}$.

Problem 3. $f_{xxy} = f_{yyx}$ if these partial derivatives exist and are continuous.

Problem 4. If $f(x,y) = \sin x + \sin y$, then $|D_{\mathbf{u}} f(x,y)| \leq \sqrt{2}$ for all unit vectors $\mathbf{u}$ and all points (x,y).

Problem 5. If f has a local minimum at (a,b) and f is differentiable at (a,b), then $\nabla f = 0$ at (a,b).

Problem 6. If f has two local maxima, then f must have a local minimum.

Problem 7. Suppose (a,b) is a critical point of f and $D = f_{xx} f_{yy} - f_{xy}^2$ has $D(a,b) > 0$ and $f_{xx}(a,b) > 0$. Then (a,b) is a minimum of $f(x,y)$ under the constraint $g(x,y) = 1$.

Problem 8.

$$\int_2^3 \int_0^1 f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_2^3 \int_0^1 f(x,y)\,\mathrm{d}y\,\mathrm{d}x.$$

Problem 9.

$$\int_0^1 \int_0^x \sqrt{x+y^2}\,\mathrm{d}x\,\mathrm{d}y = \int_0^x \int_0^1 \sqrt{x+y^2}\,\mathrm{d}y\,\mathrm{d}x.$$

Problem 10.

$$\int_3^5 \int_0^1 x^2 \sin(x^2 y^3)\,\mathrm{d}x\,\mathrm{d}y \leq 1.$$

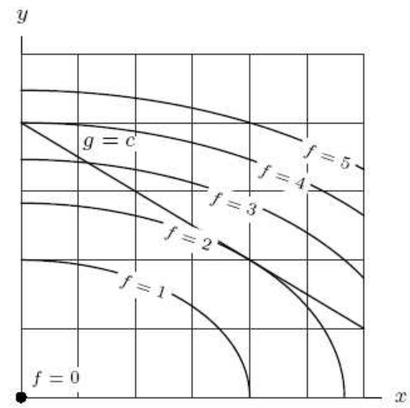
Problem 11. The average value of $f(x,y) = g(x)h(y)$ on the rectangle $R = [a,b] \times [c,d]$ is equal to the product of the average value of $g(x)$ on $[a,b]$ and the average value of $h(y)$ on $[c,d]$.

Date: Thursday, October 13.

Short answer questions.

Problem 1. Classify the function $f(x,y) = x^2y + xy$ at the origin: local max, local min, saddle point, we cannot tell, or not a critical point.

Problem 2. Find the maximum and minimum values of f on the curve $g(x,y) = c$ within the region below.



Problem 3. What is the integral of $f(x,y) = xy$ over the unit square $[0,1] \times [0,1]$?

Problem 4. Let R be the square defined by $-1 \le x, y \le 1$. What is the sign of the integral of x^4 over R? Positive, negative, zero, or cannot be determined.

Problem 5. Under what hypotheses does Fubini's theorem apply?

Problem 6. $\int_0^1 \int_0^{2-2x} f(x,y)\,\mathrm{d}y\,\mathrm{d}x$ is an integral over what region? Sketch it.

Problem 7. $\int_0^1 \int_0^{1-x} (1-x-y)\,\mathrm{d}y\,\mathrm{d}x$ computes the volume of a three-dimensional region. Sketch it.