MATH 11 FINAL "SPEED ROUND"

Set up each problem and in a few words indicate the strategy you would use to solve it. If you have time remaining, go back and solve your favorite problems.

These questions are for educational purposes. They are not necessarily indicative of what will or will not be on the final exam, nor are they indicative of how questions will be posed on the final exam.

1. Consider the vector field
$$\mathbf{F}(x, y, z) = y\mathbf{i} + (z \cos yz + x)\mathbf{j} + (y \cos yz)\mathbf{k}.$$
Show that $\mathbf{F}$ is irrotational and find a potential function for $\mathbf{F}$.

2. Express the arc length of the curve $x^2 = y^3 = z^5$ between $(1, 1, 1)$ and $(128\sqrt{2}, 32, 8)$ as an integral.

3. Calculate
$$\iint_D (x + y)^2 e^{x-y}\, dx\, dy$$
where D is the region bounded by $x + y = 1$, $x + y = 4$, $x - y = -1$ and $x - y = 1$.

4. Let $\mathbf{F} = \langle x, x^2, yz\rangle$ represent the velocity field of a fluid (measured in meters per second). Compute how many cubic meters of fluid per second are crossing the xy-plane through the square $D = \{(x, y, 0) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

5. Compute the area of the region enclosed by the hypocycloid defined by $x^{2/3} + y^{2/3} = 1$.

6. Find the points on the surface $z = 2/(xy)$ closest to the origin.

7. Find the distance from the point $(6, 1, 0)$ to the plane through the origin with normal vector $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

8. Consider
$$\mathbf{F}(x, y, z) = \langle 2x, y^2, z^2\rangle.$$
Let S be the unit sphere defined by $x^2 + y^2 + z^2 = 1$. Evaluate
$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS.$$

9. A paraboloid of revolution S is parametrized by $\mathbf{r}(u, v) = (u \cos v, u \sin v, u^2)$ over $D = \{(u, v) : 0 \leq u \leq 2, 0 \leq v \leq 2\pi\}$.
(a) Find an equation in x, y, z describing the surface.
(b) Sketch the surface. What are the geometric meanings of the parameters u, v?

(c) Find a unit vector orthogonal to the surface at $\mathbf{r}(u, v)$ as a function of u, v.
(d) Find the equation for the tangent plane at $(1, 1, 2)$ and express your answer in terms of x, y, z and parametrized by u and v.
(e) Find the surface area of S.

10. Let C be the curve defined by

$$\mathbf{r}(t) = \langle t^4/4, \sin^3(t\pi/2), e^t\rangle$$

for $0 \leq t \leq 1$. Evaluate

$$\int_C y\, dx + x\, dy.$$

11. Evaluate

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2}\, dz\, dx\, dy.$$

12. Let $f(x, y) = (x^2 + y^2)e^{-(x^2+y^2+10)}$. Find the rate of change of f at $(2, 1)$ in the direction pointing toward the origin.

13. True or false:

$$\int_1^4 \int_0^1 (x^2 + \sqrt{y}) \sin(x^2 y^2)\, dx\, dy \leq 9.$$

14. Find the x-coordinate of the center of mass of the solid region bounded by the sheet $z = 1 - x^2$ and the planes $z = 0$, $y = -1$, and $y = 1$ with a density function $\rho(x, y, z) = z(y + 2)$.

15. Find the work done by the force field

$$\mathbf{F}(x, y, z) = \langle z^3 + 2xy, x^2, 3xz^2\rangle$$

along the path C zigzagging in the plane along straight lines from $(1, 1, 1)$ to $(-1, 2, 0)$ to $(3, 0, -2)$ to $(0, -1, 1)$.