Math 11
Fall 2016
Practice Exam I

Disclaimer: This practice exam should give you an idea of the sort of questions we may ask on the actual exam. Since the practice exam (like the real exam) is not long enough to cover everything we studied, there may be topics on the real exam that are not on the practice exam, and vice versa. Anything covered in assigned reading, class, WeBWorK, or written homework is fair game.

Advice: A good way to use the practice exam is to first study and prepare for the exam. Then take a couple of hours, sit in a quiet place, and take the practice exam as if it were the real exam. That should tell you which areas you should study further.

About the real exam: There may be short answer questions that will be graded only on the answer, and there will definitely be questions on which we grade on your work, your explanations, as well as the answer.

1. TRUE or FALSE? (No partial credit; you need not show your work.)

   (a) There exists a vector $\vec{v}$ such that $\langle 2,1,2\rangle \times \vec{v} = \langle 1,-5,2\rangle$.

   (b) For any vectors $\vec{w}$ and $\vec{v}$ we have $\vec{w} \times \text{proj}_{\vec{w}}(\vec{v}) = \vec{0}$ (where $\text{proj}_{\vec{w}}(\vec{v})$ denotes the vector projection).

   (c) Any smooth parametrization of the circle $x^2 + y^2 = 1$ gives unit tangent vector at $(1,0)$ equal to $\vec{T} = \langle 0,1\rangle$.

   (d) If $\lim_{x\to 0} f(x,0) = 2$ and $\lim_{y\to 0} f(0,y) = 2$, then $\lim_{(x,y)\to(0,0)} f(x,y) = 2$.

   (e) It is possible for the intersection of the graph of f with the plane $x = 1$ to have a horizontal tangent line at $(1,2,4)$, and the intersection of the graph of f with the plane $y = 2$ to have a horizontal tangent line at $(1,2,4)$, but for the graph of f not to have a horizontal tangent plane at $(1,2,4)$.

2. Short answer questions. (No partial credit; you need not show your work.)

   (a) Determine whether the lines L_1 with parametric equations

   $$x = 6t \qquad y = 9t \qquad z = -3t + 2$$

   and L_2 with parametric equations

   $$x = -4t - 4 \qquad y = -6t + 3 \qquad z = 2t$$

   are skew, nonparallel but intersecting, parallel, or the same line.

(b) Find the arc length of the curve parametrized by $\vec{f}(t) = \langle \cos(t^2), \sin(t^2) \rangle$ for $0 \le t \le \sqrt{2\pi}$.

(c) A moving object is traveling along the parabola $y = x^2$ in $\mathbb{R}^2$, in the direction of increasing x-coordinate, at a constant speed of 2. When the object is at the point $(0, 0)$, what is its velocity?

3. Find the area of the triangle with vertices $P_0 = (-1 -1, 0)$, $P_1 = (0, 1, 1)$, $P_2 = (0, 0, 3)$.

4. Find all points (x, y) at which the graphs of the functions $f(x, y) = x^2 - 3y^2$ and $g(x, y) = 2x + 2y^3$ have parallel tangent planes.

5. Let L be the line containing the point $(1, 1, 1)$ and perpendicular to the plane defined by the equation $3x + 4y - z = 6$. Find the distance from the point $(2, 0, 0)$ to the line L.

6. A spaceship is traveling with position function $\vec{r}(t) = \left\langle \ln(t), \sqrt{2}t, \frac{t^2}{2} \right\rangle$.

   (a) How far does the spaceship travel during the time interval $\frac{1}{2} \le t \le 1$?

   (b) At time $t = 1$ the spaceship abruptly turns off its engines and continues traveling at constant velocity. Where is it located when $t = 2$?

7. Suppose that C is a level curve of the differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ and that $\vec{r} : \mathbb{R} \to \mathbb{R}^2$ is a smooth parametrization of C.

   (a) Explain why
   $$\frac{d}{dt}(f(\vec{r}(t))) = 0.$$

   (b) If $(a, b) = \vec{r}(t_0)$ is a point on C, what does $\vec{r}\,'(t_0)$ tell us about C?

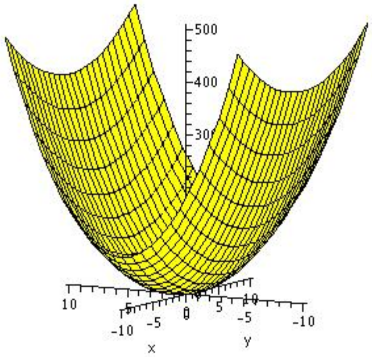
   (c) Use parts (a) and (b) and the Chain Rule to prove that if $(a, b) = \vec{r}(t_0)$ is a point on C, and if $\vec{v} = \left\langle \frac{\partial f}{\partial x}(a, b), \frac{\partial f}{\partial y}(a, b) \right\rangle$ is nonzero, then $\vec{v}$ is perpendicular to C at (a, b).

8. Each of the following three functions matches exactly one of the pictures on the next page, by either a graph or a contour plot. Identify the picture that goes with each function.

   (a) $f(x, y) = x^2 + 4y^2$

   (b) $f(x, y) = x^2 + 2xy + y^2$

   (c) $f(x, y) = 4x^2 - 2y^2$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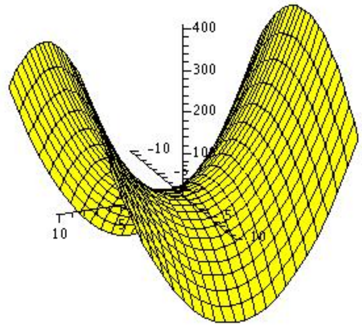
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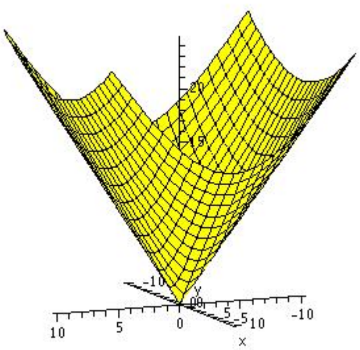
___________ **A** ____________________ **B** ____________________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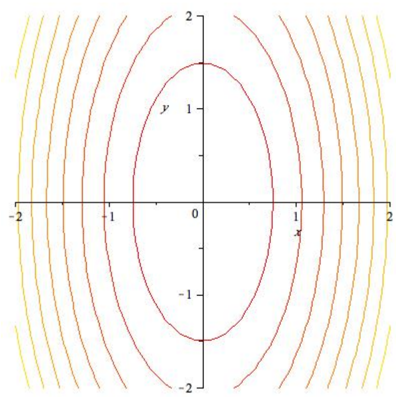
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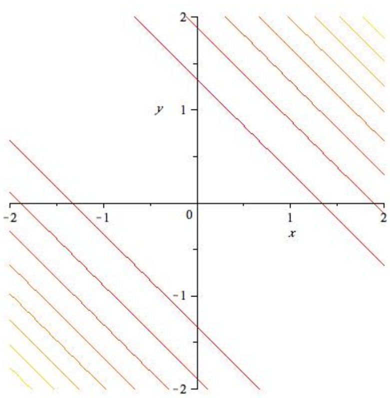
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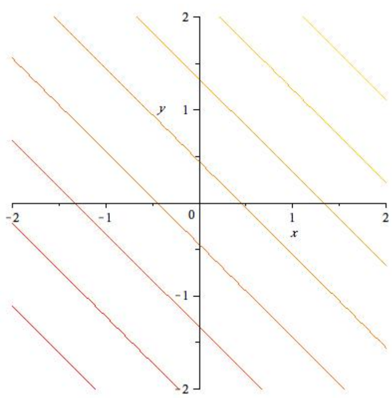
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___________ **E** ____________________ **F** ____________________