Math 11
Fall 2016
Exam II Practice Problems

1. TRUE or FALSE? (Graded on answer only; you need not show your work.)

   (a) If (a, b) is a critical point of $f(x, y)$, the second partial derivatives of f are continuous, and
   $$f_{xx}(a,b) = 2 \qquad f_{xy}(a,b) = 2 \qquad f_{yx}(a,b) = 2 \qquad f_{yy}(a,b) = 2,$$
   then (a, b) cannot be a local maximum point of f.

   (b) For any differentiable functions f and g from $\mathbb{R}^2$ to $\mathbb{R}$, we have $\nabla(f+g) = \nabla f + \nabla g$.

   (c) If f is a continuous function with continuous partial derivatives defined on the unit disc D given by $x^2 + y^2 \leq 1$, and $\nabla f(1, 0) = \langle 1, 1\rangle$, then it is possible that f attains its maximum value on D at the point $(1, 0)$.

   (d) If f is a continuous function with continuous partial derivatives and $\nabla f(0, 0) = \langle 1, 0\rangle$, then for any unit vector $\vec{u}$ we have
   $$\frac{\partial f}{\partial \vec{u}}(0,0) \leq \frac{\partial f}{\partial x}(0,0).$$

   (e)
   $$\int_0^1 \int_0^y x^2 \, dx \, dy = \int_0^y \int_0^1 x^2 \, dy \; dx.$$

2. Short answer questions. Parts (a) and (b) have nothing to do with each other. (Graded on answer only; you need not show your work.)

   (a) Rewrite
   $$\iint_D (x+y) \, dA,$$
   where D is the parallelogram with vertices $(0, 0)$, $(2, 1)$, $(3, 4)$, and $(1, 3)$, as an integral in the form
   $$\int_a^b \int_c^d f(u,v) \, du \, dv$$
   by using a suitable change of variables.

(b) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is a continuous function with continuous partial derivatives, and $\nabla f(1,2) = \langle 3, 4\rangle$.

i. What is the directional derivative of f at $(1,2)$ in the direction given by $\langle -4, 3\rangle$?

ii. What is the minimum possible value of a directional derivative $D_{\vec{u}} f(1,2)$?

3. Find the absolute maximum and absolute minimum values of

$$h(x,y,z) = x^2 + y^2 - 4x + 6y + 2z^2 - 6$$

on the region

$$R = \{(x,y) : x^2 + y^2 + z^2 = 4\}.$$

4. Find all critical points of the function $f(x,y) = x^3 - 3xy + y^3$, and classify them as local maxima, local minima, or saddle points.

5. Find

$$\iint_D xy\, dA$$

where D is the region in the first quadrant between the curves $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

6. Two surfaces S and T are given in spherical coordinates by

$$\text{surface } S: \quad \phi = \frac{\pi}{3}$$

$$\text{surface } T: \quad \rho = 4\cos\phi.$$

(a) Describe the surfaces S and T.

(b) Find the volume of the solid that lies above S and below T.

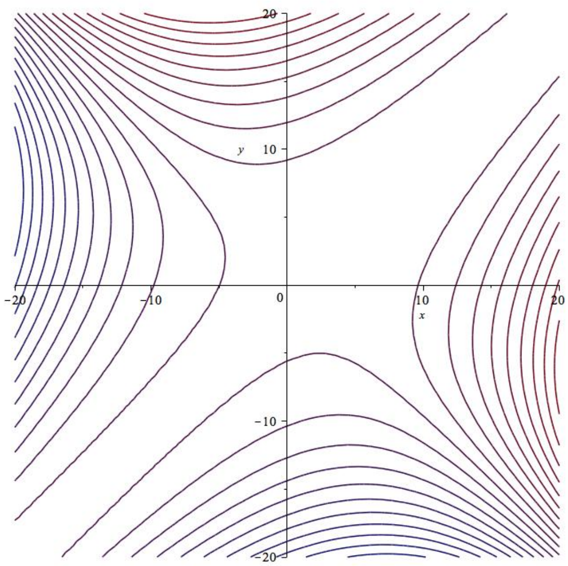
7. Sketch the region of integration, and rewrite the integral, first with the opposite order of integration, and then as an integral in polar coordinates.

$$\int_0^{\frac{1}{2}} \int_{\frac{\sqrt{3}}{2}}^{\sqrt{1-y^2}} \frac{1}{\sqrt{x^2+y^2}}\, dx\, dy.$$

8. Evaluate the triple integral

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} \sqrt{x^2+y^2}\, dz\, dy\, dx.$$

9. Short answer question. (Graded on answer only; you need not show your work.)



The picture shows the contour plot of a function $f(x, y)$. In the region $x > 5$, $y < -5$, both f_x and f_y are positive.

Does $f_{xx}(10, -15)$ appear to be positive or negative?

Does $f_{yy}(10, -15)$ appear to be positive or negative?