MATH 11: MULTIVARIABLE CALCULUS
FALL 2018
HOMEWORK #4

Please turn in your completed homework assignment by leaving it in the boxes labeled "Math 11" in the hallway outside of Kemeny 105 anytime before 3:30 p.m. on **Wednesday, October 10**.

Problem 1. Walter White's car breaks down in the middle of the desert. Walter steps out and tries to call for help, but there is almost no cell phone reception. The strength of the signal in decibel-milliwatts is given by the function

$$f(x, y, z) = \frac{(x + 2y)^2}{12e^z} - 110,$$

where (x, y, z) is the position of the phone. Trying to pick up signal, he frantically starts waving his phone around in a path given by the equation

$$\mathbf{r}(t) = \langle \sin \pi t + 2 \sin 2\pi t, \cos \pi t - 2 \cos 2\pi t, -\sin 3\pi t\rangle,$$

where t is time in seconds. Exactly 3 seconds after he started, is the strength of the received signal increasing or decreasing, and at what rate?

Problem 2. A spaceship is traveling in space when its engines fail. Without the engines the ship starts to drift towards a star. The temperature in space changes according to the function

$$T(x, y, z) = \frac{4,900}{2x^2 + y^2 + z^2}$$

The engineers are able to restart the engine when they are at position $(1, 2, 1)$.

(a) Calculate the direction in which they should proceed to cool the spaceship most rapidly. Express your answer in the form of a unit vector.
(b) At what rate (degrees per unit distance) will it cool if they go in that direction?
(c) At the present speed, the ship should not cool off at a rate that exceeds 300 degrees per unit distance. This means that the ship can not travel in the direction you found in item (a). Find the angle between the direction of maximum decrease of the temperature T and the direction in which the temperature decreases at exactly 300 degrees per unit distance. (Note: the position of the ship is still at coordinates $(1, 2, 1)$.)

Problem 3.

(a) Find all critical points of the function $f(x, y) = x^2 + y^2 + x^2y + 4$.
(b) For each critical point, determine whether it is a local maximum, a local minimum, or a saddle point.
(c) Find the absolute maximum of the function $f(x, y)$ on the domain D of points (x, y) where $0 \leq x \leq 2$ and $-2 \leq y \leq 0$.

Date: Due **Wednesday, October 10**, 3:30 p.m.