CURRENT PROBLEMS IN TOPOLOGY-MATH 124: AN INTRODUCTION TO LOW-DIMENSIONAL TOPOLOGY THROUGH FLOER THEORY

0.1. **Class and office hours.** *Class*- MWF 11:30-12:35; 242 Kemeny Hall
X-hours- Tu 12:15-1:05.
Office hours- Wed 2-3.

0.2. **Course description.** This course is designed to engage advanced undergraduate and graduate students with recent research and developments in low-dimensional topology through the lens of Floer homology. We will begin by reviewing foundational topics typically studied in low-dimensional topology, focusing on manifolds of dimension four or less. Following this, we will explore a range of exciting breakthroughs from the past decade, including subjects such as knot concordance, exotic smooth structures, exotically knotted surfaces, and exotic diffeomorphisms.

In the latter part of the course, we will go into the tools underlying these results. Students will gain an introduction to Heegaard Floer homology, an invariant of closed 3-manifolds, as well as knot Floer homology, an invariant associated with knots. Additionally, we will examine several technical aspects of the theory, including invariance, functoriality, and surgery formulas, providing a deeper understanding of these powerful methods.

0.3. **Enrollment requirement.** Although listed as a topics course, my goal is to make this accessible to strong undergraduate students and junior graduate students. If you an undergraduate student who is looking to take this course, please reach out to me. If you are a graduate student– you are eligible to take this course, provided that you have either already taken or will be taking a course in Differential Topology concurrently.

0.4. **Course evaluation.** Optional homework will be assigned to supplement the class material, but there will be no exams. The sole formal evaluation for the course will take place at the end, where students will select a research article in advance (from a given list), study it (under guidance), and present it in an informal seminar-style talk. This exercise is designed to provide insight into reading research papers and to enhance your presentation skills. Graduate students enrolled in the course are strongly encouraged to attend the topology seminars regularly for additional benefit.

0.5. **Resources.** Here is a non-exhaustive set of books which we will draw from during the first half of the course.

- Introduction To Symplectic Topology By Dusa Mcduff & Dietmar Salamon.
- Morse Theory by John Milnor.
- 4-manifolds and Kirby calculus, by Bob Gompf and Andras Stipticz
- J-holomorphic curves and Quantum cohomology, by Dusa Mcduff & Dietmar Salamon.
- Lecture notes on Morse homology with an eye towards Floer theory, by Micheal Hutchings
- Characteristic Classes by John Milnor.
- Invariants of Homology 3-Spheres by Nikolai Saveliev.
- Lectures on the Topology of 3-Manifolds: An Introduction to the Casson Invariant by Nikolai Saveliev.

Each of these books/notes could be a course of their own. In particular, you are not expected to be familiar with them. They will serve as additional reading materials in the first part of the course. If you would like to leaf through a book, start with the last book in the list. An online copy is freely available from the Dartmouth Library.