MATH 125: EXPLICIT METHODS FOR HILBERT MODULAR SURFACES WINTER 2018

JOHN VOIGHT

Course Info

- **Lectures**: Tuesday, Thursday, block 10A (10:10–12:00 noon)
- **x-period**: Wednesday, block 10AX (3:30–4:20 p.m.)
- **Dates**: 3 January 2018 – 6 March 2018
- **Room**: 343 Kemeny Hall
- **Instructor**: John Voight
- **Office**: 341 Kemeny Hall
- **E-mail**: `jvoight@gmail.com`
- **Instructor's Office Hours**: TBD, or by appointment
- **Course Web Page**: `http://www.math.dartmouth.edu/~m125w18/`
- **Prerequisites**: None
- **Required Texts**: None
- **Recommended Text**: *Quaternion algebras* (available at `http://quatalg.org`).
- **Grading**: Grade will be based on either homework or on a final project (in each case, 100%).

Syllabus

In this specialized topics course in number theory, we will compute equations for Hilbert modular surfaces.

Classical modular curves, the quotients of the upper half-plane by congruence subgroups of $\mathrm{SL}_2(\mathbb{Z})$ parametrize elliptic curves with level (torsion) structure. Equations for modular curves may be obtained by computing modular forms and multiplying their q-expansions.

One dimension up, and in an analogous way, Hilbert modular surfaces parametrize abelian surfaces with endomorphisms by an order in a real quadratic field and with level structure. In this case, it is more complicated (but by now, standard) to compute the Hecke eigenvalues for the associated Hilbert modular forms, but it is more complicated both to understand the structure of the ring of modular forms (e.g., the degrees of generators and relations), and even to multiply the corresponding q-expansions, now series indexed by totally positive elements of the inverse different of the order.

In this course, we will explore the above problem by first diving into the relevant mathematics and surveying the literature where the first few examples are worked out. Then we will design, implement, and run an algorithm to automate the generation of equations and models for Hilbert modular surfaces, including the maps between them and a description of the (stacky) universal abelian surface over them.

The course will be offered in two tracks. In one track, students who have taken previous courses in algebraic number theory, the geometry of discrete groups, modular forms, and elliptic curves, will be involved in a "final project" which is a (hopefully awesome and publishable) research paper containing what we discover. In a second track, students without this background but who still would like to learn whatever they can are invited to come to lecture and follow along; their grade will be determined by doing exercises with the background material, primarily coming from parts IV and V from the quaternion algebras book.

Homework

For those on the homework track, details about the assignments will be discussed in class and will be likely be made on an individual basis.

Late homework will be accepted but all homework must be turned in by the last day of class.

Academic Honor Principle

Cooperation on homework is permitted (and encouraged), but please write the solution up on your own and indicate on your assignment the names of any other collaborators you worked with.

Plagiarism, collusion, or other violations of the Academic Honor Principle, after consultation, will be referred to the The Committee on Standards. If you have any questions as to whether some action would be acceptable under the Academic Honor Principle, please speak to me beforehand.

Expectations

In all settings, collaborate thoughtfully and ask questions respectfully: everyone should be able to participate in the course.

Religious Observances and Accommodation

Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please meet with me before the end of the second week of the term to discuss appropriate accommodations.

I encourage students with disabilities, including "invisible" disabilities such as chronic diseases and learning disabilities, to discuss with me after class or during my office hours appropriate accommodations that might be helpful to you.

Students with disabilities enrolled in this course and who may need disability-related classroom accommodations are encouraged to make an appointment to see me before the end of the second week of the term. All discussions will remain confidential, although the Student Accessibility Services office may be consulted to discuss appropriate implementation of any accommodation requested.