Math 13, Multivariable Calculus
Written Homework 1

1. (a) Show that if f is a constant function of two variables (so $f(x, y) = k$ for some constant k) and $R = [a, b] \times [c, d]$, then

$$\iint_R k \, dA = k(b - a)(d - c).$$

   (b) Use part (a) to show that

$$0 \leq \iint_R \sin(\pi x) \cos(\pi y) \, dA \leq \frac{1}{32},$$

   where $R = [0, 1/4] \times [1/4, 1/2]$.

2. Suppose that for all x and y, $f(x, y) = f(x, -y)$. If $R = [-a, a] \times [-b, b]$ and $S = [-a, a] \times [0, b]$, what is the relationship between $\iint_R f(x, y) dA$ and $\iint_S f(x, y) \, dA$?

3. Find the average value of $f(x, y) = e^y \sqrt{x + e^y}$ over the rectangle $R = [0, 4] \times [0, 1]$.

4. Sketch the solid whose volume is given by the following iterated integral, and compute the value of the integral:

$$\int_0^1 \int_0^1 (2 - x^2 - y^2) \, dy \, dx.$$

5. Evaluate the following integral by interchanging the order of integration:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} \, dx \, dy.$$

6. In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:

$$\iint_D f(x, y) \, dA = \int_0^2 \int_0^{\sqrt{y}} f(x, y) \, dx \, dy + \int_2^4 \int_{y-2}^{\sqrt{y}} f(x, y) \, dx \, dy.$$

   Sketch the region D and express the double integral as an iterated integral with reversed order of integration.