Math 13, Multivariable Calculus
Written Homework 3

1. (Ch 15.8, #28) Find the mass of a ball B given by $x^2 + y^2 + z^2 \leq a^2$ if the density at any point is proportional to its distance from the z-axis.
   Hint: even though both cylindrical and spherical coordinates work for this problem, spherical coordinates give a simpler integral.

2. (Ch 15.9, #28) Find the average distance from a point in a ball of radius a to its center.

3. (Ch 12.4, #48) If $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$, show that
   $$\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}.$$

4. (Ch 15.10, #18) Evaluate $\iint_R (x^2 - xy + y^2)\, dA$, where R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$. Use the change of variables $x = \sqrt{2}\, u - \sqrt{2/3}\, v$, $y = \sqrt{2}\, u + \sqrt{2/3}\, v$.

5. (Ch 15.10, #14) Let R be the region bounded by hyperbolas $y = 1/x$, $y = 4/x$, and the lines $y = x$, $y = 4x$, in the first quadrant. Find equations for a transformation T that maps a rectangular region S in the uv-plane onto R, where the sides of S are parallel to the u- and v- axes.

6. (Ch 15.10, #19) Use the transformation $x = u/v$, $y = v$ to evaluate the integral $\iint_R xy\, dA$, where R is the region in the first quadrant bounded by the lines $y = x$ and $y = 3x$ and the hyperbolas $xy = 1$, $xy = 3$.