Math 13, Multivariable Calculus
Written Homework 4

1. (Ch 12.3, #54,) If $\mathbf{r} = \langle x, y, z\rangle$, $\mathbf{a} = \langle a_1, a_2, a_3\rangle$, $\mathbf{b} = \langle b_1, b_2, b_3\rangle$, show that the vector equation $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$ represents a sphere, and find its center and radius.

2. (Ch 13.2, #34) At what point do the curves $\mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2\rangle$ and $\mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2\rangle$ intersect? Find their angle of intersection. (You can write your answer as the value of an inverse trigonometric function.)

3. Find the length of the curve $\mathbf{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t\rangle$ on the interval $0 \le t \le 1$.

4. (Ch 14.3, #72) If $g(x, y, z) = \sqrt{1 + xz} + \sqrt{1 - xy}$, find g_{xyz}. (Hint: use a different order of differentiation for each term if you want to keep calculations simple.)

5. (Ch 14.4, #42) Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation for S but you know that the curves

$$\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2\rangle, \quad \mathbf{r}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1\rangle$$

both lie on S. Find an equation of the tangent plane at P.