Math 13, Multivariable Calculus
Written Homework 6

1. (Ch 16.3, #29) Show that if the vector field $\mathbf{F} = \langle P, Q, R \rangle$ is conservative and P, Q, R have continuous first-order partial derivatives, then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \qquad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}, \qquad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}.$$

2. (Ch 16.3, #35) Let $\mathbf{F} = \dfrac{-y\mathbf{i} + x\mathbf{j}}{x^2 + y^2} = \langle P, Q \rangle$.

   (a) Show that $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$.

   (b) Show that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is not independent of path. Stewart's Hint: Consider two different semicircular paths from $(1, 0)$ to $(-1, 0)$. You could also use Theorem 3 in section 16.3. Does this contradict Theorem 6?

3. (Ch 16.3, #36a) Suppose that $\mathbf{F}$ is an inverse square field; that is,

$$\mathbf{F}(\mathbf{r}) = \frac{c\mathbf{r}}{|\mathbf{r}|^3}$$

   for some constant c, where $\mathbf{r} = \langle x, y, z \rangle$. Find the work done by $\mathbf{F}$ in moving an object from a point P_1 along a path to a point P_2 in terms of the distances d_1, d_2 from these points to the origin.

4. (Ch 16.4, #2) Evaluate the line integral below by using two methods: direct evaluation and Green's Theorem, and check that the answers are identical.

$$\int_C xy\,dx + x^2\,dy,$$

   where C is the rectangle (with positive orientation) with vertices $(0, 0), (3, 0), (3, 1), (0, 1)$.

5. Verify Green's Theorem for $P(x, y) = x$ and $Q(x, y) = xy$, where D is the unit disk $x^2 + y^2 \leq 1$.

6. Compute

$$\int_C (e^{x^2}\,dx + dy),$$

   where C is the semicircle $x^2 + y^2 = 1$, $x \geq 0$ traced from $(0, -1)$ to $(0, 1)$.