Math 13, Multivariable Calculus
Written Homework 7

1. (Chapter 16.4, #22) Let D be a region bounded by a simple closed path C in the xy-plane. Use Green's Theorem to prove that the coordinates $(\bar{x}, \bar{y})$ of the centroid (the centroid is the center of mass of D, if we assume that D is a lamina of uniform density ρ and area A) of D are
$$\bar{x} = \frac{1}{2A}\int_C x^2\,dy, \quad \bar{y} = -\frac{1}{2A}\int_C y^2\,dx.$$

2. (Chapter 16.5, #20) Is there a smooth vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\nabla \times \mathbf{G} = \langle xyz, -y^2z, yz^2\rangle$? Explain.

3. (Chapter 16.5, #25.) Prove $\operatorname{div}(f\mathbf{F}) = f \operatorname{div}\mathbf{F} + \mathbf{F}\cdot\nabla f$ assuming that the appropriate partial derivatives exist and are continuous. Here $\mathbf{F} = \langle P, Q, R\rangle$ and P, Q, R, f are all scalar-valued functions of the variables x, y, z.

4. (Chapter 16.6, #24) Find a parametric representation for the surface which is the part of the sphere $x^2 + y^2 + z^2 = 16$ which lies between the planes $z = -2$ and $z = 2$.

5. (Chapter 16.6, #26) Find a parametric representation for the part of the plane $z = x+3$ that lies inside the cylinder $x^2 + y^2 = 1$.

6. (Chapter 16.6, #36) Let $\mathbf{r}(u, v) = \langle \sin u, \cos u \sin v, \sin v\rangle$. Find an equation for the tangent plane to this surface at $u = \pi/6$, $v = \pi/6$.