Math 13, Multivariable Calculus
Written Homework 9

1. (Chapter 16.7, #29) Let $\mathbf{F} = \langle x, 2y, 3z \rangle$, and let S be the cube with vertices $(\pm 1, \pm 1, \pm 1)$ with positive orientation. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

2. (Chapter 16.7, #49) Let $\mathbf{F}$ be an inverse square vector field (that is, $\mathbf{F}(\mathbf{r}) = c\mathbf{r}/|\mathbf{r}|^3$ for some constant c, where $\mathbf{r} = \langle x, y, z \rangle$). Show that the flux of $\mathbf{F}$ across a sphere S centered at the origin is independent of the radius of S.

3. (Chapter 16.9, #18) Let $\mathbf{F} = \langle z \tan^{-1}(y^2), z^3 \ln(x^2+1), z \rangle$. Find the flux of $\mathbf{F}$ across the part of the paraboloid $x^2 + y^2 + z = 2$ that lies above the plane $z = 1$ and is oriented upwards.

4. (Chapter 16.9, #24) Use the Divergence Theorem to evaluate

$$\iint_S (2x + 2y + z^2)\, dS,$$

   where S is the sphere $x^2 + y^2 + z^2 = 1$.

5. (Chapter 16.8, #17) A particle moves along line segments from the origin to the points $(1, 0, 0)$, $(1, 2, 1)$, $(0, 2, 1)$, and then back to the origin under the influence of the force field $\mathbf{F} = \langle z^2, 2xy, 4y^2 \rangle$. Find the work done in two separate ways: (a) by directly calculating this line integral, and (b) by using Stokes' Theorem with a suitable choice of surface S.

6. (Chapter 16.8, #18) Evaluate

$$\int_C (y + \sin x)\, dx + (z^2 + \cos y)\, dy + x^3\, dz,$$

   where C is the curve $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$, $0 \le t \le 2\pi$.
   (Hint: Observe that C lies on the surface $z = 2xy$.)