Math 13: Written Homework #3.
Due Wednesday, October 7.

1. Find the volume of the solid that lies between the cone $z = \sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2=2$.

2. (§15.8 #28) Find the mass of the ball B given by $x^2+y^2+z^2 \le a^2$ if the density at any point of the ball is proportional to its distance from the z-axis. (You may do the problem any way you wish, but spherical coordinates give a simpler integral.)

3. (§15.9 #28) Find the average distance of a point in a solid ball of radius a to its center.

4. (§12.4 #48) Suppose that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are vectors in $\mathbf{R}^3$ such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$. Show that $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}$.

5. (§15.10 #18) Evaluate

$$\iint_R (x^2 - xy + y^2)\, dA,$$

where R is the region bounded by the ellipse $x^2-xy+y^2=2$. Use the change of variables $x=\sqrt{2}u-\sqrt{2/3}v$ and $y=\sqrt{2}u+\sqrt{2/3}v$.

6. (§15.10 #14) Let R be the region in the first quadrant bounded by the hyperbolas $y=1/x$, $y=4/x$, and the lines $y=x$ and $y=4x$. Find the equations for the transformation T that maps a rectangular region S of the uv-plane onto R, where the sides of S are parallel to the u- and v-axes.