MATH 13, FALL ’16

HOMEWORK 3

Due Tuesday, October 4

Note: The homework is due Tuesday instead of Wednesday because the exam is on Tuesday, and I want you to do the homework before the exam. It’s an important part of preparation!

Write your answers neatly and clearly. Use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification.

Remember that you may discuss the problems with classmates, but all work should be your own. List the names of anybody with whom you discussed the problems at the top of the page.

1. Find the mass and center of mass of the solid cube \( W = [0,C] \times [0,C] \times [0,C] \) with density \( \rho(x,y,z) = x^2 + y^2 + z^2 \).

2. Find the average value of the function \( f(x,y,z) = y \) over the region in the first octant lying above \( z = 2x \) and below \( z = 1 - y^2 \).

3. Express, but do not evaluate, the integral of \( x + 2y + z \) over the region in the first octant above the surface \( z = \sqrt{x^2 + y^2} \) and below the surface \( z = \sqrt{4 - x^2 - y^2} \) in:
   a) Cartesian coordinates,
   b) cylindrical coordinates,
   c) spherical coordinates.

4. Let \( W \) be the solid region within the cylinder \( x^2 + y^2 = 1 \), above the paraboloid \( z = 1 - x^2 - y^2 \) and below the plane \( z = 2 \). Find the integral of the function \( \sqrt{x^2 + y^2} \) over this region.