Math 13 Exam #1 Review

Name: _____

Show all of your work for full credit. You will have 105 minutes to complete the exam. Remember to sketch the region when asked. Simplify if there is an obvious way to do so, but some answers are ugly and do not need to be simplified. If you run out of room for an answer, continue on the back of the page.

1 Multiple Choice

Each question in this section is worth 5 points. Indicate your choice clearly by circling the correct answer. If it is too difficult to determine which answer you chose, you will receive no credit.

1. What is the volume of the solid bounded by $z = 0$ and $z = 1 - x^2 - y^2$?

   A. $\frac{\pi}{2}$ B. π C. 2π D. $\frac{3\pi}{2}$ E. $\frac{\pi}{4}$

2. Compute the following integral:

$$\int\int_{\mathcal{R}} \frac{y}{x} dA$$

   where $\mathcal{R} = [1, 4] \times [8, 10]$.

   A. $18ln(4)$ B. $36ln(4)$ C. $18e^4$ D. $\frac{9}{2}$ E. $36e$

3. Which of the following integrals computes the mass of the tetrahedron with vertices $(0, 0, 0)$, $(2, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 4)$ and density function $\delta(x, y, z) = 3xy^2z^3$?

   A.
$$\int_0^2 \int_0^x \int_0^y 3xy^2z^3 dzdydx$$

   B.
$$\int_0^2 \int_0^{2-x} \int_0^{2-x-y} 3xy^2z^3 dzdydx$$

   C.
$$\int_0^4 \int_0^{2-\frac{x}{2}} \int_0^{2-y-\frac{x}{2}} 3xy^2z^3 dzdydx$$

   D.
$$\int_0^2 \int_0^{2-x} \int_0^{4-2x-2y} 3xy^2z^3 dzdydx$$

   E.
$$\int_0^2 \int_0^{x-2} \int_0^{2x+2y-4} 3xy^2z^3 dzdydx$$

4. Let $\mathcal{A}$ be $[-3, 3] \times [-4, 2]$, let $\mathcal{B}$ be $[-1, 2] \times [0, 2]$, and let $\mathcal{C}$ be $[-3, 3] \times [-4, 0]$. Which of the following is true?

A.
$$\int\int_{\mathcal{A}} ydA + \int\int_{\mathcal{B}} ydA = \int\int_{\mathcal{A}} ydA$$

B.
$$\int\int_{\mathcal{A}} xdA = \int\int_{\mathcal{B}} xdA = \int\int_{\mathcal{D}} xdA$$

C.
$$\int\int_{\mathcal{L}} x^3 - 1dA < 0 < \int\int_{\mathcal{A}} xdA$$

D.
$$\int\int_{\mathcal{B}} ydA = \int\int_{\mathcal{A}} ydA$$

E.
$$\int\int_{\mathcal{C}} x^3 - 1dA < 0 < \int\int_{\mathcal{B}} ydA$$

5. What is the total mass of the lamina bounded by $y = x^2$, $y = 0$, and $y = 4 - 3x$ with density function $\delta(x, y) = x$?

   A. 2 B. $\frac{2}{5}$ C. $\frac{1}{4}$ D. $\frac{3}{4}$ E. 1

6. Compute $\int\int_{\mathcal{D}} x - ydA$, where $\mathcal{D}$ is the upper half of the circle of radius 2 centered at the origin.

   A. $\frac{4\pi}{3}$ B. $\frac{-16}{3}$ C. $\frac{-4\pi}{3}$ D. -8 E. 8

7. Which of the following integrals represents the result of changing the integral
$$\int_{-1}^{-1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} x^2 + y^2 + z^2 dzdydx$$
   into spherical coordinates?

   A.
$$\int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^4 \sin\phi dr d\phi d\theta$$

   B.
$$\int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^2 \sin\phi dr d\phi d\theta$$

   C.
$$\int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^2 dr d\phi d\theta$$

   D.
$$\int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho^4 \sin\phi dr d\phi d\theta$$

   E.
$$\int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^3 dr d\phi d\theta$$

8. Which integral represents the result of changing the order of integration in the following integral?

$$\int_0^2 \int_0^{3-\frac{3x}{2}} \int_0^{6-2y-3x} x^2 + y^3 dz dy dx$$

A.

$$\int_0^2 \int_0^{6-z} \int_0^{2-\frac{2y}{3}-z} x^2 + y^3 dx dy dz$$

B.

$$\int_0^6 \int_0^{6-2z} \int_0^{3-y-\frac{z}{2}} x^2 + y^3 dx dy dz$$

C.

$$\int_0^2 \int_0^{3-2z} \int_0^{6-2y-3z} x^2 + y^3 dx dy dz$$

D.

$$\int_0^6 \int_0^{3-\frac{3z}{2}} \int_0^{6-2y-z} x^2 + y^3 dx dy dz$$

E.

$$\int_0^6 \int_0^{3-\frac{z}{2}} \int_0^{2-\frac{2y}{3}-\frac{z}{3}} x^2 + y^3 dx dy dz$$

2 Free Response

Answer each of the following questions in the space provided. Each question is worth 20 points. Show your work for full credit.

9. Compute the area of the region bounded by $y = x^2$, $y = -x^2$, $y = x^2 - 4x + 4$, and $y = 4x - x^2 - 4$.

10. Calculate the center of mass of the region bounded by $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$ with density function $\delta(x, y, z) = 2$.

11. Compute the integral

$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{1}{\cos(\theta)}} r^2 \sin(\theta) dr d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\frac{1}{\sin(\theta)}} r^2 \sin(\theta) dr d\theta$$

Hint: Describe the region and switch to Cartesian coordinates.