

Math 13 Exam #2 Review

Name: _____

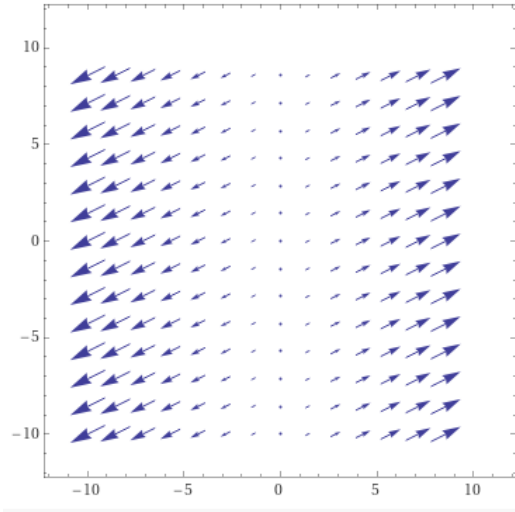
Show all of your work for full credit. You will have 105 minutes to complete the exam. Remember to sketch the region when asked. Simplify if there is an obvious way to do so, but some answers are ugly and do not need to be simplified. If you run out of room for an answer, continue on the back of the page.

1 Multiple Choice

Each question in this section is worth 5 points. Indicate your choice clearly by circling the correct answer. If it is too difficult to determine which answer you chose, you will receive no credit.

- Which of the following is the image of the v -axis under the map $G(u, v) = (e^{u+v} + 3, 2e^v + 4u)$
 - The ray $y = 2x - 6$ starting at and including the point $(3, 0)$
 - The line $y = 2x - 6$
 - The ray $y = 2x - 6$ starting at but excluding the point $(3, 0)$
 - The positive y -axis
 - The positive x -axis
- Consider the change of variables F from the xy -plane to the uv -plane for which $u = x + 2y$ and $v = 3x - y$. Let G be the inverse of F . What is the area of $G([0, 2] \times [0, 2])$?
 - 28
 - $\frac{4}{7}$
 - $\frac{7}{4}$
 - $\frac{1}{7}$
 - 4
- Let C be the curve parametrized by $r(t) = (2t^2, t^3, e^t)$ for $0 \leq t \leq 1$. Compute $\int_C F \cdot dr$, where $F(x, y, z) = \langle x, 2y, -z \rangle$.
 - $\frac{e^2}{2}$
 - $7e^2$
 - $2 + e$
 - $1 - e$
 - $\frac{7 - e^2}{2}$
- What is the length of the curve $r(t) = (2 \cos(2t), 2 \sin(2t), 2t)$, $0 \leq t \leq 2\pi$?
 - $\sqrt{5}\pi$
 - 2π
 - $\sqrt{5}\pi$
 - $4\sqrt{5}\pi$
 - 4π

5. Which of the following vector fields corresponds to the diagram below?



- A. $F(x, y) = \langle 2x, x \rangle$
- B. $F(x, y) = \langle y, 2y \rangle$
- C. $F(x, y) = \langle \cos(x), \sin(y) \rangle$
- D. $F(x, y) = \langle x^2, x^2 \rangle$
- E. $F(x, y) = \langle 0, x^2 + x + 2 \rangle$

6. Which of the following vector fields is conservative?

- A. $F(x, y) = \langle x^2, x^2 \rangle$
- B. $F(x, y) = \langle 3xy, 2 \rangle$
- C. $F(x, y) = \langle x \sin(y), y \sin(x) \rangle$
- D. $F(x, y) = \langle -y, x \rangle$
- E. None of the above

2 Free Response

Answer each of the following questions in the space provided. Questions 7 and 8 are worth 15 points, Questions 9 and 10 are worth 20 points. Show your work for full credit.

7. Compute the curl and divergence of the vector field $F(x, y, z) = \langle x^2y^2, 3xy + 2z^2, z^3x \rangle$.

8. Compute $\iint_D y^2 - x^2 dA$ using the change of variables $u = y - x$ and $v = x + y$, where D is the parallelogram with vertices $(0, 0)$, $(1, 1)$, $(-1, 1)$, and $(0, 2)$.

9. Consider the change of variables $G(u, v) = (\frac{v}{u}, \frac{u+1}{v})$.

- Give equations which describe the region $G([1, 2] \times [1, 2])$ in the xy -plane.

- Compute $|Jac(G)|$.

- Is G injective on $[1, 2] \times [1, 2]$? Justify your answer.

10. Find the potential function f with $f(1, 0, 0) = 3$ for the following conservative vector field:

$$F(x, y, z) = \langle \cos(x) \cos(y) z^3 x + \sin(x) \cos(y) z^3, -\sin(x) \sin(y) z^3 x, 3 \sin(x) \cos(y) z^2 x \rangle$$