Math 13 Final Exam Review

Name: ______________________________

Show all of your work for full credit. You will have 165 minutes to complete the exam. Remember to sketch the region when asked. Simplify if there is an obvious way to do so, but some answers are ugly and do not need to be simplified. If you run out of room for an answer, continue on the back of the page.

1 Multiple Choice

Each question in this section is worth 5 points. Indicate your choice clearly by circling the correct answer. If it is too difficult to determine which answer you chose, you will receive no credit.

1. Which of the following integrals represents the integral of $f(x, y, z) = xz + y$ over the cylinder of radius 3 of height 4 centered at the origin?

   A. $\int_0^{2\pi} \int_0^3 \int_{-2}^2 zr^2 \cos\theta + r^2 \sin\theta dzdrd\theta$

   B. $\int_0^{2\pi} \int_0^4 \int_0^3 zr \cos\theta + r \sin\theta drdzd\theta$

   C. $\int_0^{2\pi} \int_0^3 \int_0^4 zr^2 \cos\theta + r^2 \sin\theta dzdrd\theta$

   D. $\int_0^{2\pi} \int_{-2}^2 \int_0^3 zr \cos\theta + r \sin\theta drdzd\theta$

   E. None of the above

2. Let C be the closed rectangular path traversing the vertices $(0, 3)$, $(0, 6)$, $(6, 6)$, and $(6, 3)$ in that order. Calculate $\int_C F \cdot dr$, where $F(x, y) = \langle x + y, y^2 + e^{xy} \rangle$.

   A. $6e^{36} - 3e^{18}$ B. $e^{36} + 15$ C. $\frac{e^{18}}{6} - \frac{e^{36}}{6} + 21$ D. 108 E. 0

3. Let E be the solid cylinder of radius 3 and height 6 centered at the origin with density function $\delta(x, y, z) = y + 2$. What is the center of mass?

   A. $(\frac{1}{18}, \frac{1}{18}, \frac{1}{18})$ B. $(3, 6, 0)$ C. $(0, 1, 0)$ D. $(0, 0, 0)$ E. $(0, \frac{243}{216}, 0)$

4. Use the change of variables $x = u+4v$, $y = 2u-v$ to compute $\int\int_D x^2 dxdy$, where D is the parallellogram with vertices $(0, 0)$, $(3, 6)$, $(12, -3)$, and $(15, 3)$.

   A. 1 B. 243 C. -1024 D. 0 E. 5589

5. Which of the following maps $G(u, v)$ is injective on the uv-plane?

   A. $G(u, v) = (v, v + u^2)$

   B. $G(u, v) = (v^2, u^2)$

   C. $G(u, v) = (u^2, v)$

   D. $G(\cos(u), \sin(v))$

   E. None of the above

6. Let $F(x, y) = (3x^2 + 2y^2, 4xy)$ be a map from the xy-plane to the uv-plane and let G be its inverse. What is $|Jac(G)|$?

   A. $\frac{1}{24x^2+16y^2}$ B. $|24x^2 + 16y^2|$ C. 4 D. $\frac{1}{|24x^2-16y^2|}$ E. $4xy$

7. Calculate the integral of $f(x, y) = y^2 + 2x$ over the trapezoid with vertices $(0, 0)$, $(3, 0)$, and $(1, 1)$, and $(2, 1)$.

   A. $\frac{5}{2}$ B. 13 C. $\frac{13}{2}$ D. 1 E. 0

8. Let S be the surface of the tetrahedron with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, and $(0, 0, 0)$ oriented outwards. Let $F(x, y, z) = \langle 2x, 2y, 3z\rangle$. Compute the flux of F through S.

   A. -7 B. 1 C. $\frac{1}{3}$ D. $\frac{7}{6}$ E. $\frac{-3}{4}$

9. What is the length of the curve $S(t) = (t^2 + t, t^2 - t, \frac{4\sqrt{2}t^{\frac{3}{2}}}{3})$, $0 \le t \le 5$?

   A. $30\sqrt{2}$ B. 10 C. $\sqrt{30}$ D. $6\sqrt{5}$ E. 15

2 Free Response

Answer each of the following questions in the space provided. Questions 10, 11, and 12 are worth 15 points, Questions 13, 14, and 15 are worth 20 points. Show your work for full credit.

10. Calculate $\oint_C F \cdot dr$, where $F(x, y) = \langle y^3, y + x^3 \rangle$ and C is the closed triangle path which traverses the vertices $(0, 0)$, $(4, 0)$, and $(2, -2)$, in that order.

11. Compute $\int\int\int_{\mathcal{E}}(3x^2 + 4y^2)dV$, where $\mathcal{E}$ is the region bounded by $z = 0$ and $z = 1 - \sqrt{x^2 + y^2}$.

12. Is the vector field

$$F = \langle 3x^2y^3 + e^{xyz}, 3x^2y^3 + xze^{xyz}, xye^{xyz} \rangle$$

conservative? Justify your answer.

13. Find the total mass of the solid sphere E of radius 5 density function $\delta(x, y, z) = x^2 + y^2 + 2z^2$.

14. Calculate $\int\int_S F \cdot dS$, where S is the surface of the cube $[-1,1] \times [-1,1] \times [0,2]$ without the face $z = 2$ and F is the vector field $\langle x^2, y^2, z^2 \rangle$.

15. Calculate the flux of the vector field $\langle x^2, 2xy, z + y\rangle$ through the positively oriented surface parametrized by

$$R(s, t) = \langle s, t, s + t\rangle$$

with $0 \leq s \leq 4$, $0 \leq t \leq 2s$.