

Math 13 Homework #1

Due: September 22nd, beginning of class

Show all of your work for full credit. Remember to sketch the region when asked. Simplify if there is an obvious way to do so, but some answers are ugly and do not need to be simplified.

1 Iterated Integrals (R&A 15.1, OS 5.1)

1. The function $f(x, y)$ has function values given by the following table:

$y \backslash x$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{5}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{2}{3}$
$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{5}$
1	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$

Calculate the Riemann sum $S_{4,4}$ over the square $[0, 1] \times [0, 1]$ with bottom-left vertices.

2. Sketch the region $\mathcal{R} = [0, 5] \times [0, 3]$. Evaluate the integral

$$\int \int_{\mathcal{R}} (15 - 3x) dA$$

3. Sketch the region $\mathcal{R} = [2, 5] \times [4, 7]$. Evaluate the integral

$$\int \int_{\mathcal{R}} -5 dA$$

4. Calculate $\int_1^3 \int_0^2 x^3 y dy dx$.

5. Calculate $\int_0^1 \int_0^2 (x + 4y^3) dx dy$.
6. Calculate $\int_0^1 \int_2^3 \frac{1}{(x+4y)^3} dx dy$.
7. Let $\mathcal{R} = [-2, 4] \times [1, 3]$. Calculate $\int \int_{\mathcal{R}} \frac{x}{y} dA$.
8. Let $\mathcal{R} = [0, 1] \times [1, 2]$. Calculate $\int \int_{\mathcal{R}} e^{3x+4y} dA$.
9. Evaluate $\int_0^1 \int_0^1 y\sqrt{1+xy} dy dx$.

2 More General Regions (R&A 15.2, OS 5.2)

1. Sketch the domain \mathcal{D} given by $0 \leq x \leq 1$ and $x^2 \leq y \leq 4 - x^2$. Calculate the integral $\int \int_{\mathcal{D}} y dA$.
2. Sketch the domain \mathcal{D} given by $x + y \leq 12$, $x \geq 4$, and $y \geq 4$. Calculate the integral $\int \int_{\mathcal{D}} e^{x+y} dA$.
3. Calculate the integral of $f(x, y) = 2$ over the region \mathcal{D} bounded above by $y = x(2 - x)$ and below by $x = y(2 - y)$.
4. Let \mathcal{D} be defined by $\frac{1}{2} \leq x \leq \frac{\pi}{2}$ and $1 \leq y \leq 2x$. Calculate $\int \int_{\mathcal{D}} \cos(2x + y) dA$.
5. Let \mathcal{D} be bounded by $y = x - 1$, $y = 7 - x$, $y = 2$, and $y = 4$. Calculate $\int \int_{\mathcal{D}} e^{x+y} dA$.
6. Consider the iterated integral $\int_0^4 \int_x^4 f(x, y) dy dx$. Sketch the region of integration and rewrite this integral using the opposite order.
7. Find the volume of the region bounded by $z = 40 - 10y$, $z = 0$, $y = 0$, and $y = 4 - x^2$.
8. Calculate the average value of $f(x, y) = e^{x+y}$ on the square $[0, 1] \times [0, 1]$.