Math 13 Homework #2

Due: September 29th, beginning of class

Show all of your work for full credit. Remember to sketch the region when asked. Simplify if there is an obvious way to do so, but some answers are ugly and do not need to be simplified.

1 Triple Integrals (R&A 15.3, OS 5.4)

1. Integrate z^4 over the region $\mathcal{B}$ defined via $2 \leq x \leq 8$, $0 \leq y \leq 5$, $1 \leq z \leq 2$.
2. Integrate $\frac{z}{x}$ over the region $\mathcal{B} = [1,2] \times [0,3] \times [2,3]$.
3. Integrate $x + y$ over the region $\mathcal{R}$ defined via $y \leq z \leq x$, $0 \leq y \leq x^2$, $0 \leq x \leq 3$.
4. Integrate e^z over the tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,4,0)$, and $(0,0,1)$.
5. Find the bounds of integration for the region above $z = y^2$ and below $z = 8 - x^2 - y^2$.
6. Describe the region of integration for the integral

   $$\int_{-3}^{3} \int_{-\sqrt{9-z^2}}^{\sqrt{9-z^2}} \int_{-\sqrt{9-y^2-z^2}}^{\sqrt{9-y^2-z^2}} 1dxdydz$$

   Give the value for this integral without computing it directly.
7. Find the volume of the solid in the first octant bounded by the planes $x + y + z = 1$ and $x + y + 2z = 1$.
8. Set up (but do not solve) an integral which computes the volume of the region $\mathcal{W}$ bounded by the surfaces $z = 1 - y^2$, $x = 0$, $z = 0$, and $z + x = 1$.
9. Compute the average value of xyz over the region defined via $0 \leq y \leq 1 - x^2$ and $0 \leq z \leq x$.

2 Polar Coordinates (R&A 11.3, 15.4, OS 1.3, 5.3)

1. Convert the following points from Cartesian to polar coordinates.
   - $(1, \sqrt{3})$
   - $(-\sqrt{2}, -\sqrt{2})$,
   - $(0, 10)$
2. Convert the following points from polar to Cartesian coordinates.
   - $(2, \frac{\pi}{2})$,
   - $(5, \frac{\pi}{6})$
   - $(10, 0)$
3. Convert the following equations into polar coordinates.
   - $x^2 + y^2 = 9$
   - $xy = \frac{x}{y}$
   - $e^{\sqrt{x^2+y^2}} = \frac{1}{2}$
4. Convert the following equations into Cartesian coordinates.
   - $r\cos(\theta) = 0$
   - $r = \tan(\theta)$
   - $r^2 - r = \csc(\theta)$
5. For the following integral, sketch the region of integration and evaluate by changing to polar coordinates:
$$\int_0^4 \int_0^{\sqrt{16-x^2}} arctan(\frac{y}{x})dydx$$
6. For the following integral, sketch the region of integration and evaluate by changing to polar coordinates:
$$\int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}dydx$$
7. Integrate $f(x, y) = x$ over the region defined via $2 \leq x^2 + y^2 \leq 4$.
8. Compute the integral
$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{1}{\cos(\theta)}} r^2 \sin(\theta)drd\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\frac{1}{\sin(\theta)}} r^2 \sin(\theta)drd\theta$$

   Hint: Describe the region and switch to Cartesian coordinates.