Math 13 Homework #3

Due: October 14th, by midnight

Show all of your work for full credit. Remember to sketch the region when asked. Simplify if there is an obvious way to do so, but some answers are ugly and do not need to be simplified.

1 Change of Variables

1. Determine the image of the following domains under the transformation $G(u, v) = (2u, u + v)$.
   - The u- and v-axes
   - The rectangle $[1, 5] \times [2, 7]$
   - The line segment between $(3, 4)$ and $(5, 3)$
   - The triangle with vertices $(0, 1)$, $(1, 0)$, and $(1, 1)$
2. Describe, in the form $y = f(x)$, the images of the lines $u = c$ and $v = c$ under the transformation $G(u, v) = (\frac{u}{v}, u^2 - v^2)$.
3. Let $G(u, v) = (e^u, e^{u+v})$.
   - If G one-to-one? What is the image of G?
   - Describe the image of the vertical line $u = c$ and the horizontal line $v = c$.
4. Compute the Jacobian of $G(u, v) = (u + v, u - v)$. Compute the absolute value of its determinant.
5. Compute the Jacobian of $G(r, \theta) = (r\cos(\theta), r\sin(\theta)$, compute the absolute value of its determinant, and evaluate it at the point $(3, \frac{3\pi}{2})$.
6. Compute the Jacobian of $G(u, v) = (\frac{u-v}{v}, u^2 - v)$. Compute its determinant.
7. Let $\mathcal{D}$ be the parallelogram with vertices $(0, 0)$, $(4, 2)$, $(1, 6)$, and $(5, 8)$. Use the change of variables $x = 4u + v$ and $y = 2u + 6v$ to compute $\int\int_{\mathcal{D}} x^2 + ydA$.

8. Let $\mathcal{D}$ be the parallelogram with vertices $(0,0)$, (a,c), (b,d), and $(a+b,c+d)$. Give a linear transformation which sends $[0,1]\times[0,1]$ in the uv-plane to $\mathcal{D}$ in the xy-plane.

9. Let $G(u,v)=(3u+v,u-2v)$. Use the Jacobian to determine the area of $G([0,3]\times[0,5])$ and $G(\mathcal{R})$, where $\mathcal{R}$ is the triangle with vertices $(0,0)$, $(-1,9)$, and $(5,9)$.

10. Let $\mathcal{D}=\mathcal{G}([1,6]\times[1,4])$, where $G(u,v)=(\frac{u^2}{v},\frac{v^2}{u})$.

    - Compute the Jacobian of G and find the absolute value of its determinant.
    - Sketch $\mathcal{D}$.
    - Use the change of variables given by G to compute $\int\int_{\mathcal{D}} x+ydA$.

11. Use a change of variables to compute $\int\int_{\mathcal{D}} 2x-ydA$, where $\mathcal{D}$ is the parallelogram with vertics $(4,0)$, $(3,2)$, $(3,6)$, and $(4,4)$.

12. Find a mapping $G(u,v)$ which maps the circular disk $u^2+v^2\leq 1$ to the elliptical disk $(\frac{x}{a})^2+(\frac{y}{b})^2\leq 1$. Using G, calculate the area of the latter.

13. Calculate $\int\int_{\mathcal{D}} e^{4x^2+16y^2}dA$, where $\mathcal{D}$ is defined by $(\frac{x}{4})^2+(\frac{y}{2})^2\leq 1$.

14. Sketch the domain $\mathcal{D}$ bounded by $y=x^2$, $y=\frac{x^2}{2}$, and $y=x$. Use the change of variables $x=uv$ and $y=u^2$ to calculate the improper integral $\int\int_{\mathcal{D}}\frac{1}{y}dA$.

15. Let $\mathcal{D}$ be the trapezoid with vertices $(0,1)$, $(0,2)$, $(1,0)$, and $(2,0)$. Use the change of variables $u=x+y$ and $v=x-y$ to compute $\int\int_{\mathcal{D}}(x^2-2xy+y^2)^2e^{x+y}dA$.