Math 13 Homework #4

Due: October 20th, beginning of class

Show all of your work for full credit. Remember to sketch the region when asked. Simplify if there is an obvious way to do so, but some answers are ugly and do not need to be simplified.

1 Vector Fields

1. Sketch the vector field $F(x, y) = (x + 1)\hat{i} + (x - y)\hat{j}$ at the points (i, j) for integers i and j with $0 \leq i, j \leq 3$.(I.e., you need to draw sixteen vectors. Direction and relative magnitude are important, but you do not need magnitudes to be to scale.)

2. Is the vector field $F(x, y, z) = \langle \sin(x)\cos(y), \sin(x)\sin(y), \cos(x) \rangle$ a unit vector field? Justify your answer.
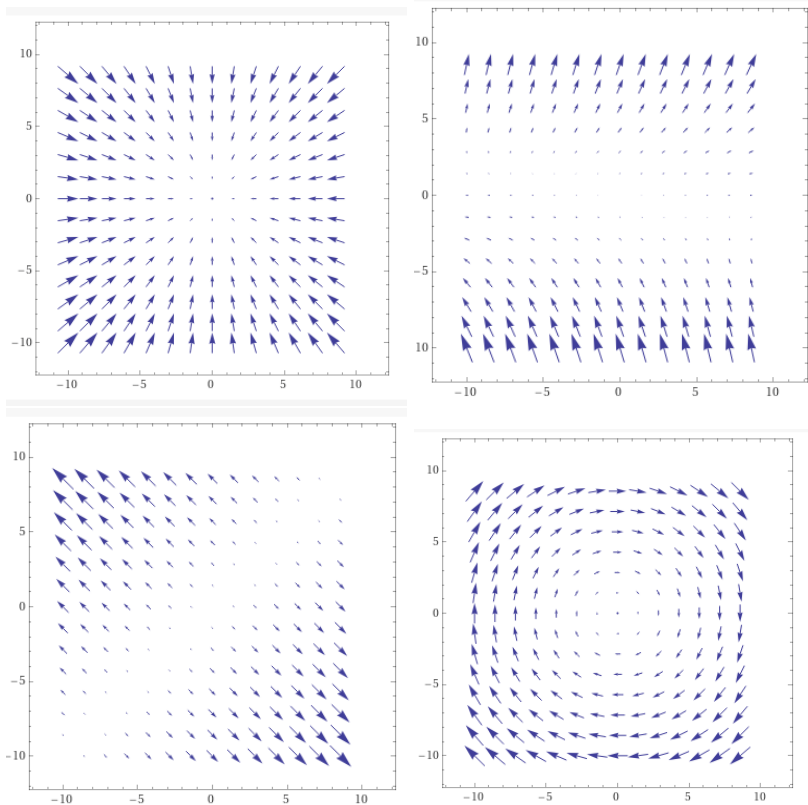
3. Match the following vector fields to their graphs.

    (a) $F(x, y) = y\hat{i} - x\hat{j}$

    (b) $G(x, y) = -x\hat{i} - y\hat{j}$

    (c) $H(x, y) = (x - y)\hat{i} + (y - x)\hat{j}$

    (d) $I(x, y) = (x + 3y)\hat{i} + y^2\hat{j}$

2 Conservative Vector Fields

1. (a) Compute the curl and divergence of the vector field $F(x, y, z) = \langle xy^2, 3z, x^2 + z^2 \rangle$.

    (b) Describe all of the sinks of F.

    (c) Describe all of the sources of F.

2. Compute a potential function for the conservative vector field $F(x, y, z) = \langle 6x^2y^2z, 4x^3yz, 2x^3y^2 \rangle$.

3. Compute a potential function for the conservative vector field $F(x, y, z) = \langle e^x z^2 + 2y, \frac{1}{y} + 2x, 2ze^x \rangle$.

4. Is the vector field $F(x, y) = \langle 3xy, 3x^2y \rangle$ conservative? Justify your answer by comparing the mixed partials.

5. Compute the curl of the vector field $F(x, y, z) = \langle 4, 5, 6 \rangle$. Is F conservative?

6. Compute the curl of the vector field $F(x, y, z) = \langle xy, zy, x^2 \rangle$. Is F conservative?

7. Is the vector field $F(x, y) = \langle 2xy \cos(x^2y), x^2 \cos(x^2y) \rangle$ conservative? Justify your answer. If it is conservative, give a potential function.

8. Is the vector field $F(x, y, z) = \langle \tan(x), \tan(y), \tan(z) \rangle$ conservative? Justify your answer. If it is conservative, give a potential function.

9. Is the vector field $F(x, y, z) = \langle e^{y+z}, e^{x+z}, e^{x+y} \rangle$ conservative? Justify your answer. If it is conservative, give a potential function.

10. Find the potential function for the conservative vector field $F(x, y, z) = \langle 3x^2y, x^3, z^2 - 2z + 1 \rangle$ such that $f(1, 2, 1) = 6$.

11. Let C be a curve between the points $(5, 18)$ and $(-1, -\pi)$. Let $F(x, y) = \langle e^x ln|y|, \frac{e^x}{y} \rangle$. Does $\int_C F \cdot dr$ depend on the path C travels?