Math 13 Homework #5

Due: November 3rd, beginning of class

Show all of your work for full credit. Remember to sketch the region when asked. Simplify if there is an obvious way to do so, but some answers are ugly and do not need to be simplified.

1 Parametric Surfaces

1. Give a parametrization of the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.
2. Give a parametrization of the part of the paraboloid $z = 4 - x^2 - y^2$ in the first octant.
3. Show that $G(r, \theta) = (r\cos\theta, r\sin\theta, 1 - r^2)$ parametrizes the paraboloid $z = 1 - x^2 - y^2$. Describe the grid curves of this parametrization.
4. Calculate T_u, T_v, and $N(u, v)$ for $G(u, v) = (2u + v, u - 4v, 3u)$ at $u = 1$, $v = 4$.
5. Calculate T_u, T_v, and $N(u, v)$ for $G(u, v) = (\cos u \sin v, \sin u \sin v, \cos v)$ at $u = \frac{\pi}{2}$, $v = \frac{\pi}{4}$.
6. Calculate T_u, T_v, and $N(u, v)$ for $G(u, v) = (u^2 - v^2, u + v, u - v)$ at $u = 2$, $v = 3$. Use $N(u, v)$ to estimate the area of the patch of $G(u, v)$ given by $2 \le u \le 2.1$ and $3 \le v \le 3.2$.

2 Surface Integrals

1. Calculate $\int\int_{\mathcal{S}} z(x^2 + y^2)dS$, where $\mathcal{S}(u, v) = (u\cos v, u\sin v, u)$ for $0 \le u \le 1$ and $0 \le v \le 1$.
2. Calculate $\int\int_{\mathcal{S}} \sqrt{x^2 + y^2}dS$, where $\mathcal{S}(u, v) = (u\cos v, u\sin v, v)$ for $0 \le u \le 1$ and $0 \le v \le 2\pi$.
3. Calculate the integral of z over the surface $\mathcal{S}$ defined via $y = 9 - z^2$ for $0 \le x \le 3$ and $0 \le z \le 3$.

4. Calculate the surface area of $\mathcal{S}$, where $\mathcal{S}$ is given by $x^2 + y^2 = 4$ and $0 \leq z \leq 4$.

5. Calculate $\int\int_S z dS$, where $\mathcal{S}$ is the part of the plane $x + y + z = 1$ in the first octant.

6. Calculate $\int\int_S \frac{z^2}{x^2+y^2+z^2} dS$, where $\mathcal{S}$ is given by $x^2 + y^2 + z^2 = 4$ and $1 \leq z \leq 2$.

7. Calculate $\int\int_S y dS$, where $\mathcal{S}$ is given by $x^2 + y^2 + z^2 = 1$ and $0 \leq y \leq 1$.

8. Use spherical coordinates to compute the surface area of a sphere of radius R.

9. Find the area of the portion of the plane $2x + 3y + 4z = 28$ which is above the rectangle $[1, 3] \times [2, 5]$ in the xy-plane.

10. Find the surface area of the part of the cone $z^2 = x^2 + y^2$ with $2 \leq z \leq 5$.

11. Calculate $\int\int_{\mathcal{S}} z e^{x+y} dS$, where $\mathcal{S}$ is the surface of the cube of side length 2 centered at the origin.