9/13 -

Course Outline

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Prereq: Math 8
Calculus Early Transcendentals Multivariable / OpenStax

%15 Daily Webwork - after each lecture, due at the start of the lecture after next. May work together.
- First due Friday before class

%15 Written Homework - Due on Wednesdays during weeks with no exam. Submitted online through Gradescope. Show work for full credit. May work together.

%20x2 Midterms - October 5th and October 26th. In person in the evenings. Submitted online through gradescope.

%30 - Final - Time and place TBD. In person submitted online via gradescope.

Tutorial - Run by Brian Mintz. Tuesday 3-6 and Friday 10-1. Online via Zoom. Link on the course webpage.

Other - For information about accessibility and accommodations, please visit the webpage. Do not hesitate to contact me with any concerns.

As with all classes at Dartmouth, masks are required in class, during exams, and in office hours.

Please attend class as long as you are feeling well. Please do not attend class if you are feeling sick.

Review

We will need to use vectors and their properties extensively.

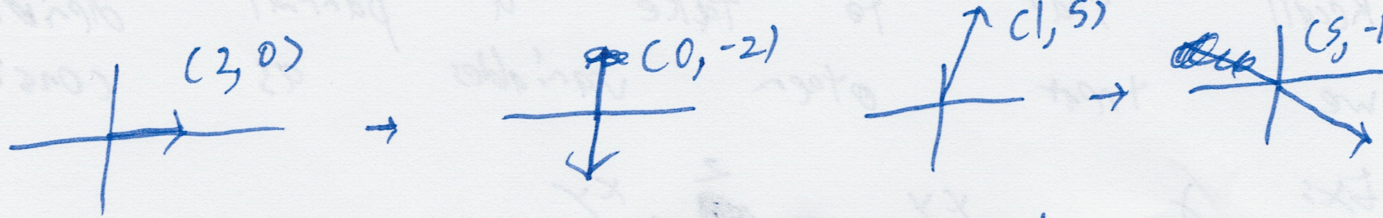
Recall that given two vectors in $\mathbb{R}^3$, $\vec{v}$ and $\vec{u}$,

$$\vec{v} \times \vec{u} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ u_x & u_y & u_z \end{bmatrix} = (v_y u_z - u_y v_z)\hat{i} - (v_x u_z - u_x v_z)\hat{j} + (v_x u_y - u_x v_y)\hat{k}$$

$\vec{v} \times \vec{u}$ is a vector perpendicular to both $\vec{v}$ and $\vec{u}$ whose magnitude is the area of the parallelogram defined via $\vec{u}$ and $\vec{v}$.

Note: Given $n-1$ vectors in $\mathbb{R}^n$, we can find a vector perpendicular to all of them using this method.

Ex: $\begin{pmatrix}\hat{i} & \hat{j}\\ x & y\end{pmatrix} = y\hat{i} - x\hat{j}$,



We can use this to find a normal vector for a plane in $\mathbb{R}^3$: Take two vectors inside the plane and compute their cross product to get a normal vector. Then we only need a point in the plane to compute a plane equation.

Ex: Find the equation of the plane containing the triangle with vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

Sol: The vectors $(-1,1,0)$ and $(-1,0,1)$ are inside the plane.

$$(-1,1,0) \times (-1,0,1) = \begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & 0\\ -1 & 0 & 1\end{pmatrix} = \hat{i} - (-\hat{j}) + \hat{k}$$
$$= (1,1,1)$$

Thus the plane equation is $x+y+z=d$.
As $(1,0,0)$ is a point in the plane, the plane equation is $x+y+z=1$.

We will also be using partial derivatives. Recall that to take a partial derivative, we treat other variables as constants.

Ex:

1) $\frac{\partial}{\partial x} ye^{xy} = y^2 e^{xy}$

2) $\frac{\partial}{\partial x} xe^{xy} = yxe^{xy} + e^{xy}$

The gradient vector ∇f of $f(x,y,z)$ at (a,b,c) is the vector $\langle f_x(a,b,c), f_y(a,b,c), f_z(a,b,c)\rangle$.

The directional derivative of f in the direction of unit vector $\langle u,v,w\rangle$ at (a,b,c) is given by $\nabla f \cdot \langle u,v,w\rangle$.

Critical points are where $\nabla f = \vec{0}$.

Given a critical point (a,b), if the second partials exist, then

$$D = f_{xx}(a,b)\, f_{yy}(a,b) - f_{xy}(a,b)^2$$

can be used to classify critical points. $D>0$, max / min

$D<0$, saddle point