2nd Major Use of Stokes Theorem

- Works on non-closed surfaces, curves.

- Particularly useful when $\nabla \cdot F = 0$ or $\nabla \times F = \langle 0, 0, 0 \rangle$.

- Essentially it works the following way: If S is a surface and T is a surface such that $S \cup T$ is closed, and V is the volume whose boundary is $S \cup T$ then we have:

$$\int \int_S \mathbf{F} \cdot \mathbf{dS} = \int \int \int_V \nabla \cdot F \; dV - \int \int_T \mathbf{F} \cdot \mathbf{dS}$$

- The technique also works with curves: If C_1 and C_2 are curves such that $C_1 \cup C_2$ is a closed curve, then if S is a surface whose boundary is $C_1 \cup C_2$ then: $\int_{C_1} F(r(t))|r'(t)|dt = \iint_S \nabla \times F \cdot \mathbf{dS} - \int_{C_2} F(r(t))|r'(t)|dt$

- In the standard notation [and the one used in the book] this second is:

$$\int_{C_1} F \cdot \mathbf{dr} = \iint_S \nabla \times F \cdot \mathbf{dS} - \int_{C_2} F \cdot \mathbf{dr}$$