

WEDNESDAY MARCH 26TH LECTURE NOTES

1. VECTORS

- Vectors have a magnitude and direction
- We most often represent vectors using the format $\langle x, y, z \rangle$ or $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.
- The magnitude, or length, of a vector in the $\langle x, y, z \rangle$ format can be easily found to be [using pythagorean theorem] $\sqrt{x^2 + y^2 + z^2}$

2. DOT PRODUCT

- The dot product, $\mathbf{a} \cdot \mathbf{b}$ of vectors \mathbf{a} , \mathbf{b} is always equal to $|a||b|\cos(\theta)$, where θ is the angle between the vectors. Note that the dot product is always a number, rather than a vector.
- If \mathbf{a} , \mathbf{b} are perpendicular, $\mathbf{a} \cdot \mathbf{b} = 0$.
- if \mathbf{a} , \mathbf{b} are parallel, $\mathbf{a} \cdot \mathbf{b} = |a||b|$.
- $|\mathbf{a}|^2 = \mathbf{a} \cdot \mathbf{a}$
- $\langle x_1, y_1, z_1 \rangle \cdot \langle x_2, y_2, z_2 \rangle = x_1x_2 + y_1y_2 + z_1z_2$

3. CROSS PRODUCT

- The cross product of two vectors is **always** perpendicular to both.
- The magnitude of the cross product is equal to the area of the parallelogram determined by the vectors.
- The direction of the cross product can be ascertained by using the "right-hand rule."
- The cross product can also be calculated as $\langle a, b, c \rangle \times \langle d, e, f \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & b & c \\ d & e & f \end{vmatrix}$