Practice Exam 1

Problem 1: Let $f(x, y) = 2 + \ln(x^2 + y^2)$. Sketch at least 3 level curves of f in the xy-plane. Sketch the graph of f.

Problem 2: Let $\mathbf{f}(x, y) = (x^2 y, 5x - 3y^2)$ and let $\mathbf{g}(s, t) = (s^3 + t^2, \frac{s}{t})$.
a. Calculate $D\mathbf{f}$ and $D\mathbf{g}$.
b. Use part a and the Chain Rule to calculate $D(\mathbf{f} \circ \mathbf{g})(1, 1)$.

Problem 3: Assume the surface of a mountain is given by the graph of a differentiable function. If the slope heading due east from the point a is 1/3 and the slope heading due north from that point is -1/3, in what direction should you head to descend most rapidly?

Problem 4: The voltage V in a circuit slowly decreases as the battery wears out. The resistance R slowly increases as the resistor heats up. Use Ohm's Law, $V = IR$, to find how the current I is changing at the moment when $R = 400\ \Omega$, $I = .08\ A$, $\frac{dV}{dt} = -.01\ V/s$, and $\frac{dR}{dt} = .03\ \Omega/s$.

Problem 5: Let $\mathbf{F}(x, y, z) = (-z, y^2, x)$.
a. Compute curl $\mathbf{F}$.
b. Verify that $\mathbf{x}(t) = (3\cos t, \frac{1}{2-t}, 3\sin t)$ is a flow line of $\mathbf{F}$.

Problem 6:
a. Describe the following region in $\mathbb{R}^2$ using polar coordinates: the region inside the circle $x^2 + y^2 = 2$ and to the right of the line $x = 1$.
b. Describe the following solid region in $\mathbb{R}^3$ using spherical coordinates: the region lying above the xy-plane, inside the sphere $x^2 + y^2 + z^2 = 4$ and outside the cone $z = \sqrt{x^2 + y^2}$.

Problem 7: Consider the following sum of integrals.

$$\int_0^{\sqrt{2}/2} \int_{-\sqrt{1-y^2}}^{\sqrt{2}/2} 2y\ dxdy + \int_{\sqrt{2}/2}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 2y\ dxdy$$

Switch the order of integration to turn this sum of two double integrals into a single double integral and evaluate it.

Problem 8: Set up an iterated integral which gives the volume of the region bonded by the parabaloid $y = 4x^2 + 4z^2$ and the plane $y = 4$.

Problem 9: Consider the cone $z^2 = x^2 + y^2$. Show that for any point (a, b, c) on the cone other than $(0, 0, 0)$, the tangent plane at the point (a, b, c) intersects the origin.