1. The two equations are given by

$$z = x^2 + y^2$$

$$(x-1)^2 + y^2 = 1$$

Use shifted cylindrical coordinates

$$\begin{cases} x = 1 + r\cos\theta \\ y = r\cdot\sin\theta \\ z = z \end{cases}$$

The integration becomes

$$\int_{r=0}^{1}\int_{\theta=0}^{2\pi}\int_{z=0}^{1+r^2+2\cos\theta\cdot r} r\cdot dz\,d\theta\,dr = \int_0^1\int_0^{2\pi} r(1+r^2+2\cos\theta\cdot r)\cdot dz\,d\theta\,dr$$

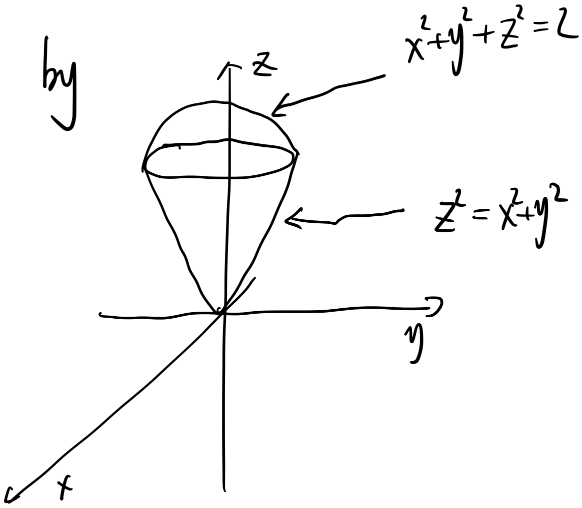
$$= 2\pi\cdot\int_0^1 (r+r^3)\,dr = \frac{3}{2}\pi$$

2. ① The range is given by

$$0 \le x \le 1$$

$$0 \le y \le \sqrt{1-x^2}$$

$$\sqrt{x^2+y^2} \le z \le \sqrt{2-x^2-y^2}$$



Using spherical coordinates, (θ, φ, ρ)

$$x^2+y^2+z^2 = 2 \implies \rho = \sqrt{2}$$

$$z^2 = x^2+y^2 \implies \varphi = \frac{\pi}{4}$$

The integration becomes

$$\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{2}}\int_0^{\frac{\pi}{4}} \rho\sin\varphi\cos\theta \cdot \rho\sin\varphi\sin\theta \cdot \rho^2\sin\varphi \, d\varphi\, d\rho\, d\theta$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\sqrt{2}}\int_0^{\frac{\pi}{4}} \rho^4 \sin^3\varphi \cdot \sin\theta\cos\theta \, d\varphi\, d\rho\, d\theta$$

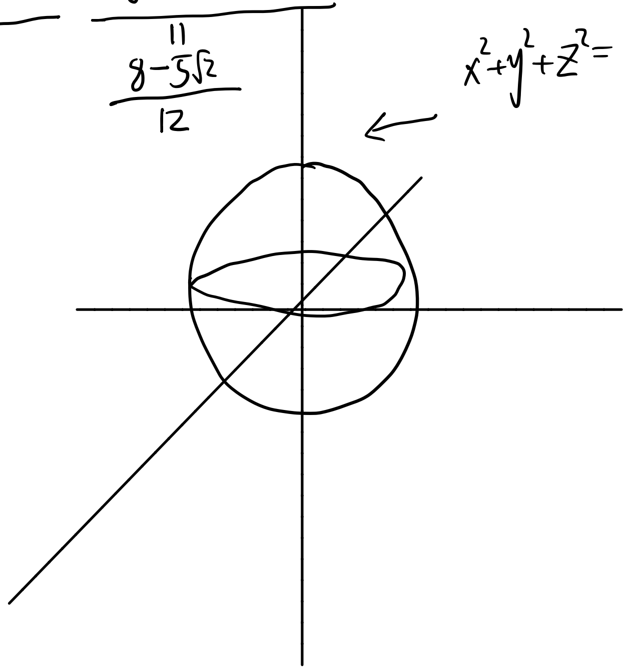
$$= \underbrace{\int_0^{\sqrt{2}} \rho^4 \, d\rho}_{\frac{4\sqrt{2}}{5}} \cdot \underbrace{\int_0^{\frac{\pi}{2}} \sin\theta\cdot\cos\theta\, d\theta}_{\frac{1}{2}} \cdot \underbrace{\int_0^{\frac{\pi}{4}} \sin^3\varphi\, d\varphi}_{\frac{8-5\sqrt{2}}{12}} = \frac{4\sqrt{2}-5}{15}$$

(2) $-a \le x \le a$

$-\sqrt{a^2-x^2} \le y \le \sqrt{a^2-x^2}$

$-\sqrt{a^2-x^2-y^2} \le z \le \sqrt{a^2-x^2-y^2}$

$x^2+y^2+z^2=a^2$



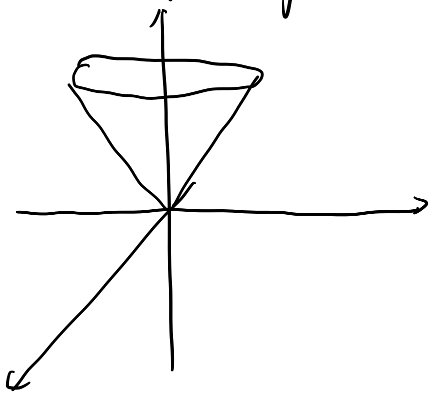
The integration:

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{a} [\rho^2\sin^2\varphi\cdot\cos^2\theta\cdot\rho\cos\varphi + \rho^2\sin^2\varphi\sin^2\theta\,\rho\cos\varphi + \rho^3\cos^3\varphi)\,\rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$$

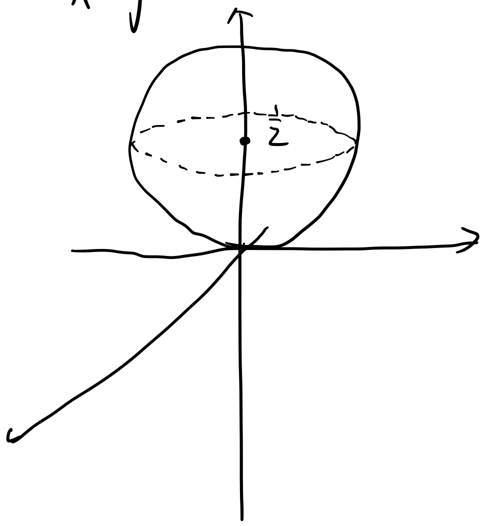
$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho\cos\varphi\cdot\rho^2\cdot\rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} d\theta \underbrace{\int_0^{\pi} \sin\varphi\cos\varphi\, d\varphi}_{0} \cdot \int_0^{a} \rho^5\, d\rho = 0$$

3. $z = \sqrt{x^2 + y^2}$



$x^2+y^2+z^2 = z$



Use cylindrical coordinates.

$$\begin{cases} x = r\cos\theta \\ y = r\cdot\sin\theta \\ z = z \end{cases} \Rightarrow$$

$z = \sqrt{x^2+y^2} \Rightarrow z = r$

$x^2+y^2+z^2 = z \Rightarrow z^2 - z + r^2 = 0 \Rightarrow z = \frac{1+\sqrt{1-4r^2}}{2}$

The intersection

$2r^2 - r = 0 \Rightarrow r = \frac{1}{2}$

Let $u = (\frac{1}{4} - r^2)$

$du = -2r\cdot dr$

The volumn

$$\int_0^{2\pi}\int_0^{\frac{1}{2}}\int_r^{\frac{1+\sqrt{1-4r^2}}{2}} r\cdot dz\, dr\cdot d\theta$$

$$= 2\pi\cdot\int_0^{\frac{1}{2}} r\cdot\left[\frac{1}{2} + \frac{1}{2}\cdot\sqrt{1-4r^2} - r\right]\cdot dr$$

$$= 2\pi\left[\frac{1}{2}\int_0^{\frac{1}{2}} r\cdot dr - \int_0^{\frac{1}{2}} r^2\cdot dr + \int_0^{\frac{1}{2}}\sqrt{\frac{1}{4}-r^2}\cdot r\cdot dr\right]$$

$$= 2\pi\cdot\left[\frac{1}{2}\cdot\frac{1}{8} - \frac{1}{3}\cdot\frac{1}{8} + \int_0^{\frac{1}{4}}\sqrt{u}\cdot\frac{1}{2}\cdot du\right] = 2\pi\cdot\left[\frac{1}{48} + \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{1}{8}\right]$$

$$= \frac{\pi}{8}$$