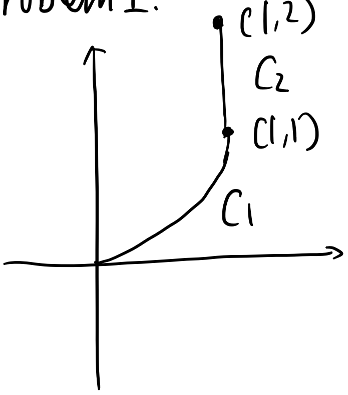
Problem 1:



We parametrize C_1 by

$$x = x, \quad y = x^2 \qquad 0 \le x \le 1$$

$$\int_{C_1} 2x\, ds = \int_0^1 2x\sqrt{1+4x^2} = \frac{1}{4}\cdot\frac{2}{3}(1+4x^2)^{\frac{3}{2}}\Big|_0^1 = \frac{5\sqrt{5}-1}{6}$$

For C_2, we parametriz by

$$x=1, \quad y = y \qquad 1 \le y \le 2$$

$$\int_{C_2} 2x \cdot ds = \int_1^2 2\cdot\sqrt{0+1^2} = \int_1^2 2\cdot dy = 2$$

Thus.

$$\int_C 2x\, ds = \frac{5\sqrt{5}-1}{6} + 2.$$

Problem #2.

(1) $$\int_C F\cdot dr = \int_0^1 \langle t^7, -t^6\rangle \cdot \langle 3t^2, 2t\rangle \cdot dt$$

$$= \int_0^1 (3t^9 - 2t^7)\cdot dt = \frac{3}{10} - \frac{1}{4} = \frac{6}{20} - \frac{5}{20} = \frac{1}{20}$$

(2)

$$\int F \cdot dr = \int_0^1 \langle \sin t^3, -\cos t^2, t^4 \rangle \cdot \langle 3t^2, -2t, 1 \rangle \, dt$$

$$= \int_0^1 \sin t^3 \cdot 3t^2 \, dt + \int_0^1 \cos t^2 \cdot 2t \, dt + \int_0^1 t^4 \, dt$$

$$= 1 - \cos 1 + \sin 1 + \frac{1}{5} = \frac{6}{5} + \sin 1 - \cos 1$$

(3).

Because $\operatorname{curl}(F) = 0$, and the field is defined on $\mathbb{R}^3$, which is simply connected. it is conservative.

Suppose

$$\begin{cases} \dfrac{\partial f}{\partial x} = y^2 \\ \dfrac{\partial f}{\partial y} = 2xy + e^{3z} \\ \dfrac{\partial f}{\partial z} = 3y \cdot e^{3z} \end{cases}$$

$$\Rightarrow f(x,y,z) = y^2 \cdot x + h(y,z)$$

$$\Rightarrow 2xy + \frac{\partial h}{\partial y} = 2xy + e^{3z} \Rightarrow \frac{\partial h}{\partial y} = e^{3z} \Rightarrow h(y,z) = e^{3z} \cdot y + g(z)$$

$$\Rightarrow f(x,y,z) = y^2 x + e^{3z} y + g(z)$$

$$\Rightarrow \frac{\partial f}{\partial z} = 3y \cdot e^{3z} + g'(z) = 3y e^{3z} \Rightarrow g'(z) = 0 \Rightarrow g(z) = C$$

$\Rightarrow$

$f(x,y,z) = y^2 x +$ [redacted] $y e^{3z} + C$.