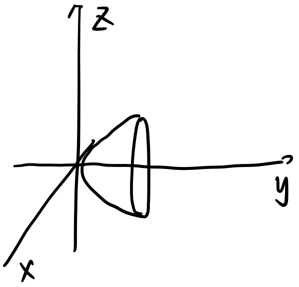
1.



$\Sigma = \Sigma_1 + \Sigma_2$, where $\Sigma_1 : y = x^2 + z^2 , \ 0 \le y \le 1$

$\Sigma_2 : x^2 + z^2 \le 1, \quad y = 1$

For Σ_1, $G(r, \theta) = (r\cos\theta, \ r, \ r\cdot\sin\theta)$

$\Rightarrow \dfrac{\partial G}{\partial r} = (\cos\theta, 1, \sin\theta), \quad \dfrac{\partial G}{\partial \theta} = (-r\cdot\sin\theta, \ 0, \ r\cos\theta)$

Normal vector

$$\frac{\partial G}{\partial \theta} \times \frac{\partial G}{\partial r} = \begin{pmatrix} i & j & k \\ -r\sin\theta & 0 & r\cos\theta \\ \cos\theta & 1 & \sin\theta \end{pmatrix} = (-r\cos\theta, \ r, \ -r\sin\theta).$$

$$\iint_{\Sigma_1} F\, ds = \int_0^1 \int_0^{2\pi} \langle 0, \ r, -r\sin\theta \rangle \cdot \langle -r\cos\theta, \ r, \ -r\sin\theta \rangle \cdot d\theta\, dr$$

$$= \int_0^1 \int_0^{2\pi} (r^2 + r^2 \sin^2\theta)\, d\theta\, dr = \int_0^1 \int_0^{2\pi} \left(\frac{3r^2}{2} - \frac{r^2\cos 2\theta}{2}\right) d\theta\, dr = \frac{r^3}{2}\Big|_0^1 \cdot 2\pi = \pi$$

For Σ_2, $G(r, \theta) = (r\cos\theta, 1, r\sin\theta)$

Normal vector $(0, r, 0)$

$$\iint_{\Sigma_2} F \cdot ds = \int_0^1 \int_0^{2\pi} \langle 0, \ r, -\sin\theta \rangle \cdot \langle 0, 1, 0 \rangle \cdot d\theta\, dr = 2\pi \int_0^1 r \cdot dr = \pi$$

$$\Rightarrow \iint_{\Sigma} F\, ds = \iint_{\Sigma_1} F\, ds + \iint_{\Sigma_2} F\, ds = 2\pi$$

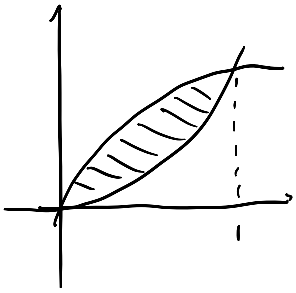
2. By Green's thm.

$$\oint_C (3y - e^{\sin x})\,dx + (7x + \sqrt{y^4+1})\,dy$$

$$= \iint_D (7-3)\,dS = 4\iint_D 1\,ds = 4\cdot\pi\cdot 3^2 = 36\pi$$

$D:\ x^2+y^2 \le 9$

3.



$$\int_C (y + e^{\sqrt{x}})\,dx + (2x + \cos y^2)\,dy$$

$$= \iint_D (2-1)\,ds = \int_0^1 \int_{x^2}^{\sqrt{x}} 1\,dy\,dx = \int_0^1 (\sqrt{x} - x^2)\,dx$$

$$= \left(\frac{2}{3}x^{\frac{2}{3}} - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{1}{3}$$