Triple Integrals

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Math13

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Double Integral Practice Problems

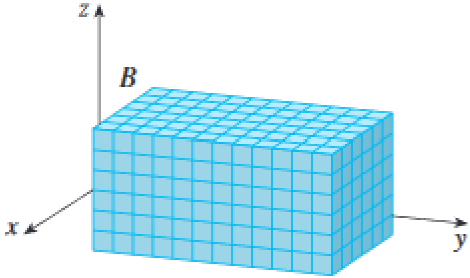
1. Integrate $f(x, y) = x$ over the region bounded by $y = x^2$ and $y = x + 2$.

2. Sketch the domain of integration for $\int_0^4 \int_x^4 f(x, y) dy dx$, and then express as an iterated integral in the opposite order.

3. Find the volume of the region bounded by $z = 50 - 10y$, $z = 10$, $y = 0$, and $y = 4 - x^2$.

Challenge Problems

1. Let $\mathcal{D}$ be the domain bounded by $y = x^2 + 1$ and $y = 2$. Prove the inequality $\frac{4}{3} \leq \iint_{\mathcal{D}} (x^2 + y^2) dA \leq \frac{20}{3}$.
2. Verify the Mean Value Theorem for $f(x, y) = e^{x-y}$ on the triangle bounded by $y = 0$, $x = 1$, and $y = x$.
3. Is it true that $\iint_{\mathcal{D}} f(x)g(y) dy dx = \left(\int_a^b f(x) dx \right) \left(\int_{h_1(a)}^{h_2(b)} g(y) dy \right)$ for vertically simple regions? Why or why not?
4. Use integrals to calculate the volume of a cone of base radius r and height h.


z
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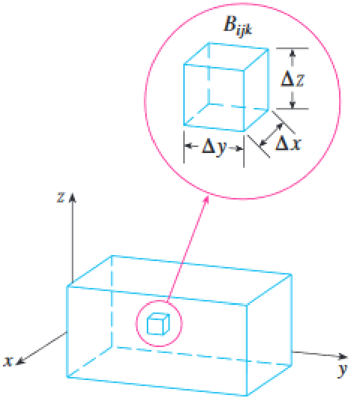

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Triple Integral Problems

4. Evaluate $\iiint_{\mathcal{B}} \frac{x}{(y+z)^2} dV$ for the box $\mathcal{B} = [0,2] \times [2,4] \times [-1,1]$.

5. Set up the triple integral $\iiint_{\mathcal{W}} f(x,y,z) dV$ where $\mathcal{W}$ is the region in the first octant above $z = y^2$ and below $z = 8 - 2x^2 - y^2$.

Challenge Problems

1. Find the volume of the region contained in the intersection of the cylinders $x^2 + y^2 \leq a^2$ and $x^2 + z^2 \leq a^2$.
2. Prove that $\int_0^x \int_0^t F(u) du dt = \int_0^x (x-u) F(u) du$.