Vector Surface Integrals

Melanie Dennis

Dartmouth College
Math13

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Dartmouth

Vector Surface Integrals Practice Problems

1. Let $\mathcal{S}$ be the rectangle $0 \leq y \leq 2$, $0 \leq z \leq 3$ in the (y, z) plane with the normal pointing in the negative x direction. Find $\iint_{\mathcal{S}} \langle \sin(y), \sin(z), yz \rangle \cdot d\mathbf{S}$.

2. Find $\iint_{\mathcal{S}} \langle x, y, e^z \rangle \cdot d\mathbf{S}$ where $\mathcal{S}$ is the cylinder $x^2 + y^2 = 4$ with $1 \leq z \leq 5$ and an outward pointing normal.

Challenge Problems

1. Let $\mathcal{S}$ be the cone $z^2 = x^2 + y^2$ with $x^2 + y^2 \leq 4$, $z \geq 0$ with a downward-pointing normal. Find $\iint_{\mathcal{S}} \langle xy, y, 0 \rangle \cdot d\mathbf{S}$.
2. Prove that if $\mathcal{S}$ is the part of a graph $z = g(x, y)$ lying over a domain $\mathcal{D}$ in the xy-plane with normal pointing upward, then

$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = \iint_{\mathcal{D}} \left(-F_1 \frac{\partial g}{\partial x} - F_2 \frac{\partial g}{\partial y} + F_3 \right) dx dy.$$