Divergence Theorem

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Divergence Theorem Practice Problems

1. Evaluate $\iint_{\mathcal{S}} \langle xy^2, yz^2, zx^2 \rangle \cdot d\mathbf{S}$ where $\mathcal{S}$ is the boundary of the cylinder $x^2 + y^2 \leq 4$, $0 \leq z \leq 3$.

2. Let $\mathbf{F} = \langle x, y, z \rangle$ and $\mathcal{W}$ be a region with a smooth boundary $\mathcal{S}$. Show that Volume$(\mathcal{W}) = \frac{1}{3} \iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$.

3. Let $\mathcal{W}$ be the region bounded by $x + 2y + 4z = 12$ and the coordinate planes in the first octant. Set up (don't evaluate) the triple integral to find the flux of $\left\langle x^2 - z^2, e^{z^2} - \cos(x), y^3 \right\rangle$ out of $\mathcal{W}$.

Challenge Problems

1. Use Problem 2 from above to find the volume of the unit ball.
2. Let $\mathcal{W}$ be the pyramid with vertices $(0,0,1)$, $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(1,1,0)$. Find the flux of $\langle x^2y, 3y^2z, 9z^2x \rangle$ out of $\mathcal{W}$.