Final Review

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Math13

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Change of Variables

1. Find an appropriate transformation $G(u, v)$
2. Find your new domain $\mathcal{D}^*$
3. Find the scaling factor (the Jacobian)
4. Plug in to the change of variable equation:

$$\iint_{\mathcal{D}} f(x, y)dA = \iint_{\mathcal{D}^*} f(G(u, v))|\text{Jac}(G)|dudv$$

Line Integrals

1. Find an appropriate parametrization $\mathbf{r}(t)$
2. Find your new domain $a \leq t \leq b$
3. Find the scaling factor $(\mathbf{r}'(t))$
4. Plug in to the line integral equation:

$$\int_{\mathcal{C}} f(x,y,z)dr = \int_a^b f(\mathbf{r}(t))||\mathbf{r}'(t)||dt$$

$$\int_{\mathcal{C}} \mathbf{F}(x,y,z) \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)dt$$

Surface Integrals

1. Find an appropriate parametrization $G(u, v)$
2. Find your new domain $\mathcal{D}$ (called parameter domain)
3. Find the scaling factor ($\mathbf{N}(u, v)$)
4. Plug in to the surface integral equation:

$$\iint_{\mathcal{S}} f(x, y, z)dS = \iint_{\mathcal{D}} f(G(u, v))||\mathbf{N}||dudv$$

$$\iint_{\mathcal{S}} \mathbf{F}(x, y, z) \cdot d\mathbf{S} = \iint_{\mathcal{D}} \mathbf{F}(G(u, v)) \cdot \mathbf{N}dudv$$

Green's Theorem

$$\int_{\mathcal{C}} F_1 dx + F_2 dy = \iint_{\mathcal{D}} \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA$$

where $\mathcal{C}$ is the (closed) boundary of $\mathcal{D}$ and is oriented such that, when walking around $\mathcal{C}$, the shape $\mathcal{D}$ is on your left.

Stokes' Theorem

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = \iint_{\mathcal{S}} \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

where $\mathcal{C}$ is the (closed) boundary of $\mathcal{S}$ and is oriented such that, when walking around $\mathcal{C}$ with your head pointed in the direction of the normal to the surface, the surface $\mathcal{S}$ is on your left.

Divergence Theorem

$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = \iiint_{\mathcal{W}} \mathsf{div}(\mathbf{F}) dV$$

where $\mathcal{S}$ is the (closed) boundary of $\mathcal{W}$ oriented with outward pointing normals.