Math 13, Multivariable Calculus
Practice problems Integration

1. Evaluate the following integral:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy.$$

2. In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:

$$\iint_D f(x,y)\, dA = \int_0^2 \int_0^{\sqrt{y}} f(x,y)\, dx\, dy + \int_2^4 \int_{y-2}^{\sqrt{y}} f(x,y)\, dx\, dy.$$

   Sketch the region D and express the double integral as an iterated integral with reversed order of integration.

3. Evaluate the triple integral $\iiint_T xyz\, dV$, where T is the solid tetrahedron with vertices $(0,0,0), (1,0,0), (1,1,0), (1,0,1)$.

4. Sketch the solid whose volume is given by the following iterated integral, and compute the value of that volume:

$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy.$$

5. Let E be the three-dimensional region lying below the plane $z = 3 - 2y$ and above the paraboloid $z = x^2 + y^2$.

   (a) Sketch the projections onto the xy- and yz-planes.

   (b) Sketch a typical cross section parallel to the xz-plane (with y constant).

   (c) Sketch the region E.

   (d) Set up the limits of integration (but do not integrate!) for the integral

$$\int\int\int_E f(x,y,z)\, dV$$

   with respect to $dzdxdy$ and $dxdydz$.