

1. (10 points) Find the volume of the first-octant region outside the paraboloid $z = x^2 + y^2$ and inside the cylinder $x^2 + y^2 = 4$.
2. Set up the following integrals. You need *not* evaluate the integrals, just set them up.

(a) (8 points) $\int_0^1 \int_{e^x}^e xy \, dy \, dx$ rewritten as an iterated integral with the x -integration performed first.

(b) (8 points) $\iiint_{\Omega} z \, dx \, dy \, dz$ rewritten in spherical coordinates, where Ω is the

region above the cone $z = \sqrt{\frac{x^2 + y^2}{3}}$ and inside the sphere $x^2 + y^2 + z^2 = 9$.

(c) (9 points) $\int_0^1 \int_x^{\sqrt{2-x^2}} \int_0^{x^2+y^2} \sqrt{x^2 + y^2} \, dz \, dy \, dx$ rewritten in cylindrical coordinates.

3. (10 points) Let $f: \mathbf{R}^3 \rightarrow \mathbf{R}$ be a differentiable function. Let $a = \frac{\partial f}{\partial x}(1, 0, -2)$,

$b = \frac{\partial f}{\partial y}(1, 0, -2)$, $c = \frac{\partial f}{\partial z}(1, 0, -2)$. Let $g: \mathbf{R}^2 \rightarrow \mathbf{R}^3$ by

$g(u, v) = (e^v + 2u, u + v, u - 2)$. Compute the derivative matrix $D(f \circ g)(0, 0)$ in terms of a , b , and c .

4. (10 points) Prove that the path $c(t) = (\cos t, \sin t, 0)$ is a flow line of the vector field

$\mathbf{F}(x, y, z) = (-y, x, z^5)$. (Be sure to state carefully what it means for a path to be a flow line of a vector field.)

6. (10 points) Find an equation for the tangent plane at the point $(-2, 1, -3)$ to the ellipsoid

given by $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.

7. (10 points) Evaluate the surface integral $\iint_{\partial W} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (1, 1, z(x^2 + y^2))$

and W is the solid cylinder given by $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$. [Hint: Do this the easy way!]

8. (10 points) Let H be the regular hexagon in the plane whose vertices are $(1, 0)$,

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $(-1, 0)$, $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. Evaluate the line integral

$\oint_{\partial H} \mathbf{F} \cdot ds$, where \mathbf{F} is the vector field $\mathbf{F}(x, y) = \left(\frac{y^2}{2}, xy + x\right)$. [Hint: Do this the easy way!]